

Research Article

Learning-Assisted Model Reference Adaptive Control for Uncertain First-Order Systems: A Five-Scenario Comparative Validation Study

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**ABSTRACT**

In this paper, a detailed computational analysis of a Learning-Assisted Model Reference Adaptive Controller (LA-MRAC) for uncertain first-order dynamic systems is presented. The proposed controller is the classical MRAC structure with an online learning compensation term $u_{ML}(t)$ to enhance the tracking performance under uncertainty, external disturbances, measurement noise and abrupt plant changes. Five increasingly difficult scenarios are developed to avoid depending on a single experiment: nominal operating, time varying parameters, external disturbances, measurement noise, and abrupt parameter change. The proposed LA-MRAC is compared to the PID controller and classical MRAC and NN-Only learning controller using a number of quantitative measures such as RMSE, MAE, IAE, ISE, ITSE, control effort, dynamic response indices, disturbance indices, and computational execution time. The revised presentation is organized in a journal-form experimental form: before introducing each table and figure, and after using them, they are interpreted. The results show that LA-MRAC consistently achieves the lowest tracking error across all scenarios. Average RMSE is reduced by 4.79% compared with classical MRAC, 37.78% compared with PID, and 62.62% compared with NN-Only control. The improvement is obtained with control effort close to classical MRAC, while the computational time increases due to the online learning component. These results support the conclusion that the proposed hybrid adaptive-learning structure provides an effective accuracy-robustness trade-off for uncertain computer-controlled dynamic systems.

1. INTRODUCTION

Adaptive control is an important research direction in computer engineering, intelligent control systems, embedded computing, and digital control applications [1]. Many practical dynamic systems operate under uncertain or changing conditions, where the mathematical model of the plant may not be completely known. In addition, real control systems may be affected by external disturbances, measurement noise, abrupt parameter variations, and unmodeled dynamics. Under these conditions, fixed-gain controllers may fail to maintain accurate tracking performance because their parameters are selected in advance and do not automatically adapt to changes in the system behavior [2-3].

The Proportional–Integral–Derivative (PID) controller is one of the most widely used control methods because of its simplicity and ease of implementation. However, its performance strongly depends on proper gain tuning and may degrade when the system parameters vary with time or when unexpected disturbances occur [4]. For systems that require higher adaptability, Model Reference Adaptive Control (MRAC) provides an attractive solution. The main concept of MRAC is to define a desired reference model that represents the required system response, and then adapt the controller parameters online so that the plant output follows the reference model output as closely as possible. This makes MRAC suitable for uncertain dynamic systems and digital control applications [5-6].

Despite its advantages, classical MRAC still has some limitations. The adaptation process may be slow when the system experiences rapid parameter changes. The controller may also produce residual tracking error under external disturbances or measurement noise [7]. Moreover, the performance of MRAC is sensitive to the selection of adaptation gains. If the adaptation gain is too small, the controller may respond slowly; if it is too large, oscillations or numerical instability may occur. These challenges motivate the development of enhanced adaptive control structures that can improve tracking accuracy and robustness while maintaining acceptable computational cost [8-9].

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In recent years, learning-based control methods have received increasing attention because neural networks and online learning algorithms can approximate unknown nonlinearities and compensate for unmodeled uncertainties [10]. However, using learning alone may not be sufficient for control tasks because a pure learning-based controller may lack the stabilizing structure required for reliable reference tracking. Therefore, combining adaptive control with learning-based compensation can provide a useful balance between stability-oriented control and data-driven uncertainty compensation [11-13].

This paper proposes a Learning-Assisted Model Reference Adaptive Controller (LA-MRAC) for uncertain first-order dynamic systems. The proposed method preserves the adaptive structure of classical MRAC and adds an online learning compensation term. The MRAC component is responsible for maintaining the reference-model tracking objective, while the learning component is used to compensate for residual uncertainty, disturbance effects, and dynamic variations that are not fully handled by the classical adaptive law. In this way, the proposed controller does not replace MRAC with a neural model; instead, it enhances MRAC using a compact learning-assisted mechanism.

The proposed LA-MRAC is evaluated against three baseline controllers: PID, classical MRAC, and NN-Only control. The NN-Only controller is included to show whether learning alone can provide reliable tracking without the MRAC adaptive structure. Five simulation scenarios are considered in order to provide a comprehensive evaluation: nominal operation, time-varying parameters, external disturbances, measurement noise, and abrupt parameter changes. These scenarios are selected to test the controller under normal, uncertain, noisy, disturbed, and sudden-change conditions.

The performance of the compared controllers is evaluated using several quantitative indicators, including Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Integral Absolute Error (IAE), Integral Square Error (ISE), Integral Time Square Error (ITSE), control effort, dynamic response indicators, disturbance rejection measures, and computational execution time. These metrics allow the proposed controller to be assessed from different aspects, including tracking accuracy, robustness, control energy, and computational cost.

The main contributions of this study are summarized as follows:

- Proposing a compact Learning-Assisted MRAC (LA-MRAC) for uncertain first-order systems.
- Adding an online learning compensation term to improve classical MRAC tracking performance.
- Comparing the proposed method with PID, classical MRAC, and NN-Only controllers.
- Evaluating the controllers under Five scenarios: nominal case, time-varying parameters, disturbances, measurement noise, and abrupt parameter changes.
- Assessing performance using tracking error, control effort, robustness, and computational time metrics.
- Demonstrating that LA-MRAC provides better tracking accuracy and robustness with acceptable computational cost.

2. RELATED WORK

Model Reference Adaptive Control (MRAC) is widely used for uncertain dynamic systems because it updates controller parameters online to make the plant output follow a desired reference model. However, classical MRAC may suffer from slow adaptation and reduced performance under disturbances, measurement noise, and sudden parameter variations. Therefore, recent studies have focused on enhancing MRAC using learning-based and neural adaptive mechanisms [14-16].

Montanaro et al. proposed a learning-enhanced MRAC strategy using a cost-effective educational plant based on DC motor speed control. Their results showed that enhanced MRAC can improve tracking performance under uncertain dynamics and disturbances. This work is close to our study because both consider first-order-like uncertain systems and aim to improve classical MRAC through an additional adaptive mechanism.[17]

Liu et al. introduced a model reference neural adaptive controller with a multilateral learning mechanism for uncertain nonlinear systems. Their approach uses neural learning to compensate for uncertainties while maintaining the model-reference tracking objective. This supports the idea of the proposed LA-MRAC, where an online learning compensation term is added to classical MRAC to improve tracking accuracy.[18]

Cheng et al. developed a state predictor-based deep MRAC for quadrotor trajectory tracking. Their method uses deep neural networks to handle unstructured uncertainties and disturbances. Although their application is more complex than the first-order system considered in this paper, their work supports the use of learning models inside MRAC structures to improve robustness under uncertain conditions.[19]

Gong et al. proposed a self-organizing MRAC method with enhanced persistent excitation for aircraft systems. Their approach uses fast adaptation and experience replay to improve response speed and robustness. This is related to our work because it shows that modern MRAC research is moving toward adaptive-learning structures that enhance performance under changing dynamics.

Based on these studies, the present work proposes a compact Learning-Assisted MRAC structure for uncertain first-order dynamic systems. The proposed method combines the adaptive tracking ability of classical MRAC with an online learning

compensation term and is evaluated against PID, classical MRAC, and NN-Only controllers under five different simulation scenarios.

3. PROPOSED METHOD

3.1 Overview of the Proposed LA-MRAC

The uncertain first-order plant is modeled as $\dot{x}(t) = -a(t)x(t) + b(t)u(t) + d(t)$, where $x(t)$ is the system output, $u(t)$ is the control input, $a(t)$ and $b(t)$ are uncertain or time-varying parameters, and $d(t)$ is an external disturbance. The desired behavior is defined by the reference model $\dot{x}_m(t) = -a_m x_m(t) + b_m r(t)$, where $r(t)$ is the reference input. The tracking error is $e(t) = x(t) - x_m(t)$.

The proposed controller uses the hybrid control law $u(t) = \theta_1(t)x(t) + \theta_2(t)r(t) + u_{ML}(t)$. The first two terms correspond to the adaptive MRAC structure, while $u_{ML}(t)$ is generated using an online learning estimator. In this way, LA-MRAC attempts to preserve MRAC behavior while adding a learning-based correction for residual uncertainty.

3.2 Compared Controllers

Table I is introduced to define the role of each controller before the experimental results are presented. This table clarifies that the comparison is not limited to a single baseline; instead, it evaluates conventional, adaptive, learning-only, and hybrid approaches.

TABLE I. CONTROLLERS CONSIDERED IN THE COMPARATIVE STUDY.

Controller	Role in the study	Main purpose
PID	Simple low-cost baseline	Reference comparison for conventional control
Classical MRAC	Adaptive control baseline	Measures the added value of learning compensation
NN-Only	Learning-only baseline	Tests whether learning alone is sufficient
Proposed LA-MRAC	Hybrid adaptive-learning proposed method	Tests the proposed combination of MRAC and online learning

As shown in Table I, the comparison structure is designed to isolate the contribution of each element. PID evaluates simplicity, classical MRAC evaluates adaptive structure, NN-Only evaluates isolated learning compensation, and LA-MRAC evaluates the proposed hybrid strategy.

4. SIMULATION SCENARIOS AND EVALUATION METRICS

Table II presents the five scenarios used to validate the proposed controller. The scenarios are arranged from easier to harder conditions so that the robustness of the controller can be evaluated progressively.

TABLE II. SIMULATION SCENARIOS USED FOR VALIDATION.

Scenario	Condition	Purpose
S1	Nominal case	Baseline tracking verification
S2	Time-varying parameters	Evaluate adaptation under drifting dynamics
S3	External disturbances	Evaluate disturbance rejection
S4	Measurement noise	Evaluate robustness to noisy measurements
S5	Abrupt parameter change	Evaluate response to sudden model mismatch

The scenario design in Table II ensures that the evaluation is not dependent on a single favorable case. The fifth scenario is the most demanding because it combines noise, disturbances, and abrupt parameter variation.

Table III lists the quantitative metrics used throughout the paper. These metrics cover tracking accuracy, accumulated error, control cost, dynamic behavior, disturbance rejection, and computational cost.

TABLE III. EVALUATION METRICS USED IN THE STUDY.

Metric	Interpretation	Preferred direction
RMSE	Root-mean-square tracking error	Lower
MAE	Average absolute tracking error	Lower
IAE	Accumulated absolute tracking error	Lower
ISE	Accumulated squared tracking error	Lower
ITSE	Time-weighted squared error	Lower
Control Effort	Integral of squared control input	Lower if accuracy is preserved
Max $ u $	Maximum required control magnitude	Lower if feasible

Metric	Interpretation	Preferred direction
Rise/Settling/Peak Time	Dynamic response speed and overshoot	Lower settling/rise with acceptable overshoot
Disturbance RMSE/IAE	Error during disturbance intervals	Lower
Execution Time	Computational cost of simulation	Lower

The use of multiple metrics in Table III is essential because a controller may reduce tracking error while increasing control effort or computational time. Therefore, the discussion does not claim absolute superiority in every category; instead, it evaluates the trade-off between accuracy, robustness, effort, and cost.

5. OVERALL QUANTITATIVE RESULTS

Before analyzing each scenario individually, the average behavior across all five scenarios is summarized. Table IV reports the main tracking metrics and control effort averaged by method. This table provides the first quantitative evidence for whether the proposed controller consistently reduces the tracking error.

TABLE IV. AVERAGE PERFORMANCE ACROSS THE FIVE SCENARIOS.

Method	RMSE	MAE	IAE	ISE	ITSE	Control Effort	Execution Time (s)	Average Time per Step (ms)
Classical MRAC	0.120683	0.079247	0.792470	0.146703	0.300810	37.267	0.115409	0.011541
NN-Only	0.307352	0.237700	2.3770	0.953246	2.5709	29.474	0.195004	0.019500
PID	0.184666	0.155521	1.5552	0.359114	2.0391	27.789	0.115091	0.011509
Proposed LA-MRAC	0.114897	0.074812	0.748121	0.132996	0.280199	37.298	0.205658	0.020566

Note. Values are averaged over the five simulation scenarios; lower values are generally preferred for the reported metrics. The results in Table IV show that Proposed LA-MRAC has the lowest average RMSE, MAE, IAE, ISE, and ITSE. Classical MRAC is the closest competitor, which confirms that the adaptive structure is already strong. However, LA-MRAC further reduces tracking error because the online learning compensation term assists the adaptive law in estimating residual uncertainty. The table also shows that LA-MRAC requires higher computational time than PID and classical MRAC, which is expected because the learning term adds operations at every simulation step.

Table V reports the average percentage improvement of LA-MRAC over each baseline method. This table directly quantifies the performance gain and avoids relying only on visual interpretation.

TABLE V. AVERAGE PERCENTAGE IMPROVEMENT OF LA-MRAC OVER BASELINE CONTROLLERS.

Baseline	RMSE	IAE	ISE	ITSE
PID	37.781	51.896	62.966	86.258
Classical MRAC	4.7946	5.5963	9.3433	6.8520
NN-Only	62.617	68.527	86.048	89.101

Note. Positive values indicate that LA-MRAC reduced the corresponding error metric compared with the baseline controller.

As shown in Table V, the improvement over classical MRAC is moderate but consistent, while the improvement over PID and NN-Only is much larger. This is scientifically important because it shows two things: first, the learning term improves classical MRAC; second, learning compensation alone is not sufficient, as NN-Only performs substantially worse than the hybrid controller.

Table VI provides a scenario-wise ranking based on tracking accuracy. Rank 1 denotes the best method in each scenario. This table is used as an additional validation tool to determine whether LA-MRAC performance is stable across scenarios.

TABLE VI. RANKING-BASED VALIDATION ACROSS THE FIVE SCENARIOS.

Scenario	Classical MRAC	NN-Only	PID	Proposed LA-MRAC
S1_Nominal_Case	2.0000	4.0000	3.0000	1.0000
S2_Time_Varying_Parameters	2.0000	4.0000	3.0000	1.0000
S3_External_Disturbances	2.0000	4.0000	3.0000	1.0000
S4_Measurement_Noise	2.0000	4.0000	3.0000	1.0000
S5_Abrupt_Parameter_Change	2.0000	4.0000	3.0000	1.0000

Note. Rank 1 represents the best tracking accuracy in the corresponding scenario.

Table VI indicates that Proposed LA-MRAC ranks first in all five scenarios, followed by classical MRAC. This supports the claim that the proposed method is not only strong on average but also consistently strong under each individual operating condition.

6. OVERALL VISUAL INTERPRETATION

To quantitatively assess tracking accuracy across the full validation set, Figure 1 presents the RMSE comparison among PID, classical MRAC, NN-Only, and Proposed LA-MRAC for all five scenarios.

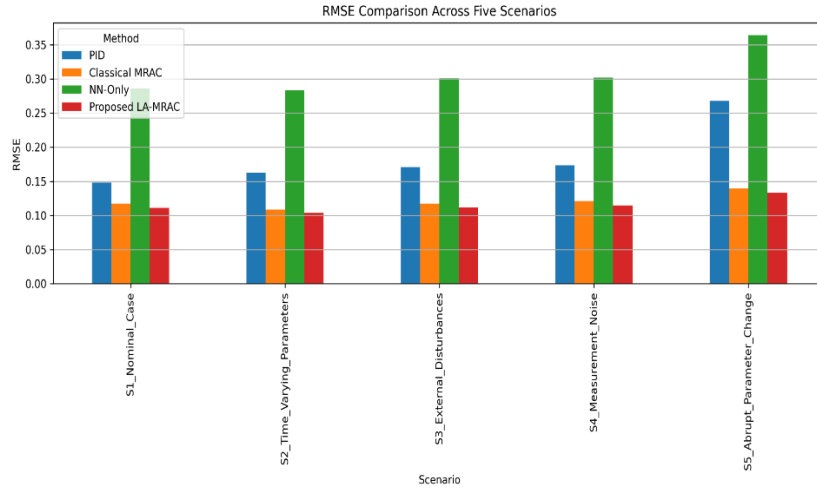


Fig. 1. RMSE comparison across the five scenarios

As can be seen in Figure 1 the proposed LA-MRAC yields the lowest RMSE in all scenarios. NN-Only records the highest RMSE indicating that learning compensation is not enough. For several cases, PID gives better performance than NN-Only, but fails to be as accurate as the adaptive controllers. As shown by the continuous decrease of RMSE, MRAC tracking accuracy under nominal and non-nominal condition is enhanced by the learning term.

The accumulated absolute tracking error is evaluated in all the scenarios through Figure 2, which displays the IAE values of all controllers.

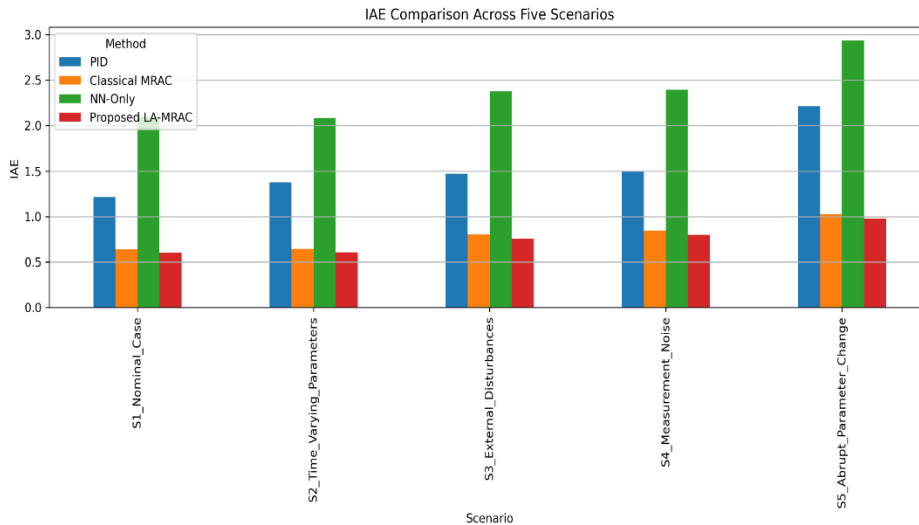


Fig. 2. IAE comparison across the five scenarios

Figure 2 confirms the same tendency observed in the RMSE comparison. LA-MRAC has the smallest accumulated error in every scenario, which means the proposed controller does not only reduce instantaneous peaks but also reduces error accumulation over the complete simulation time. Since improved accuracy can sometimes be obtained by excessive control action, Figure 3 compares the control effort for all methods and scenarios.

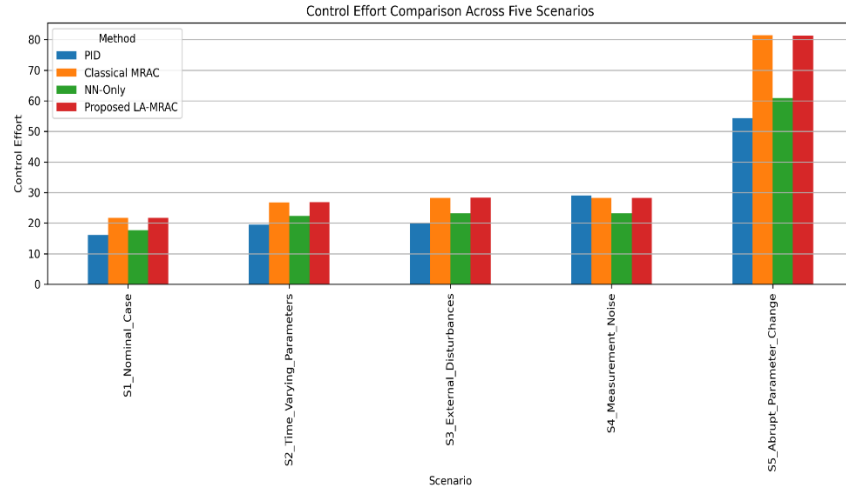


Fig. 3. Control effort comparison across the five scenarios

Figure 3 shows that LA-MRAC and classical MRAC have very close control effort values. In the abrupt-change scenario, both adaptive methods require larger effort because the plant dynamics change sharply. The key point to take away from this is that there is only a modest increase in control effort over classical MRAC, but the accuracy is improved. This shows that the accuracy-effort trade off is favourable. To examine computational feasibility, Figure 4 presents the execution time required by each controller under the five scenarios.

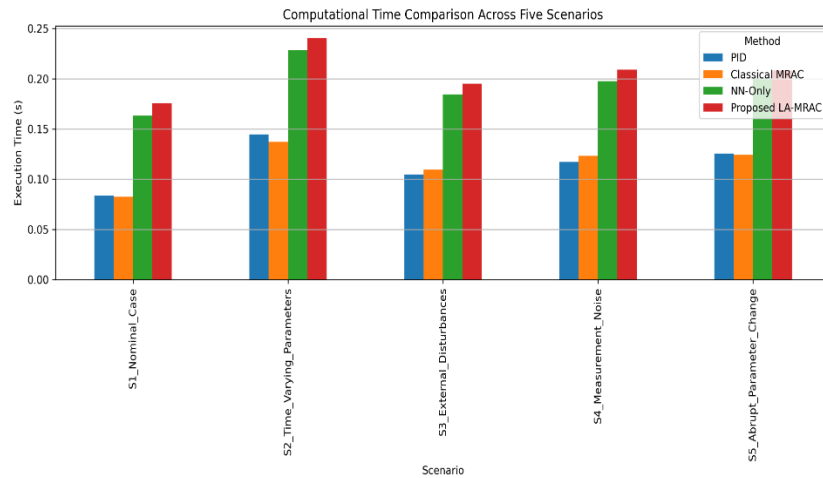


Fig. 4. Computational time comparison across the five scenarios

As can be seen in figure 4, the PID and classical MRAC are faster than the NN-Only and LA-MRAC. This is to be expected since both adaptive parameters and learning weights are updated by the proposed controller. This is a limitation but is justified with the aim of achieving better accuracy and robustness. The overall averages of each controller for all the scenarios are summarized in Figure 5.

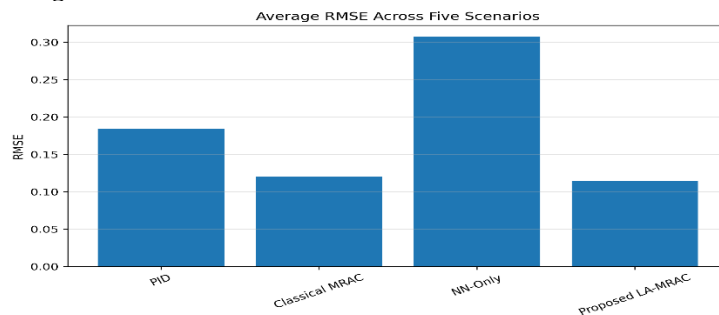


Fig. 5. Average RMSE across the five scenarios

The information in Table 4 is summarized visually in Figure 5. The average RMSE of LA-MRAC is the lowest and followed by classical MRAC. The figure is helpful to report the overall tracking superiority of the proposed method in terms of tracking accuracy. The average IAE values for all scenarios are shown in figure 6.

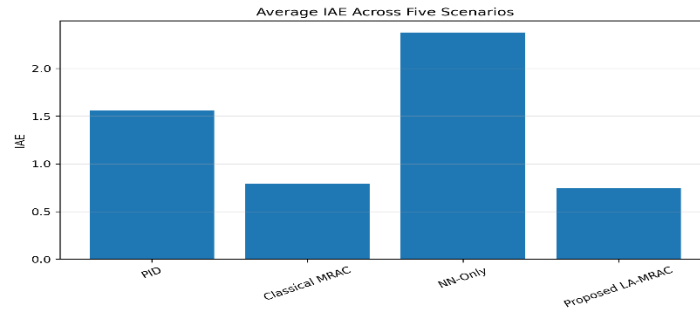


Fig. 6. Average IAE across the five scenarios

From the accumulated error analysis shown in Fig. 6, it is observed that LA-MRAC has least long-term accumulated error. The difference between LA-MRAC and NN-Only is remarkable, particularly so as they come from different viewpoints, highlighting that the adaptive structure is essential to stable tracking. Figure 7 shows a comparison of average ISE values for a squared tracking-error comparison.

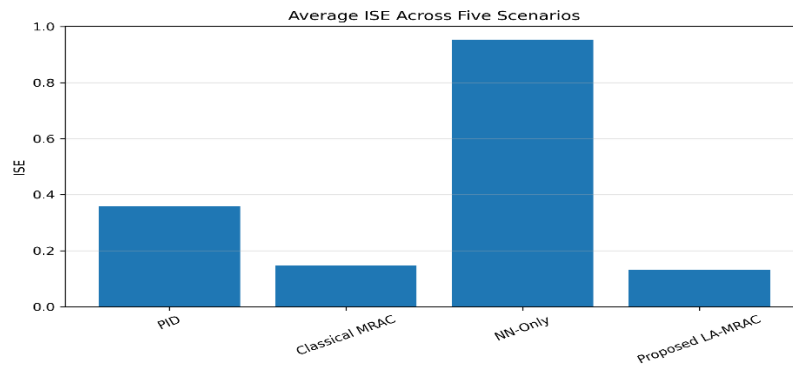


Fig. 7. Average ISE across the five scenarios

Finally, Figure 7 shows that LA-MRAC provides a decrease in squared error over all the baseline approaches. This result suggests that the proposed method is effective in suppressing the large tracking deviations, which is desirable because large errors are given greater weight in ISE. The average ITSE values for the compared methods are shown in figure 8.

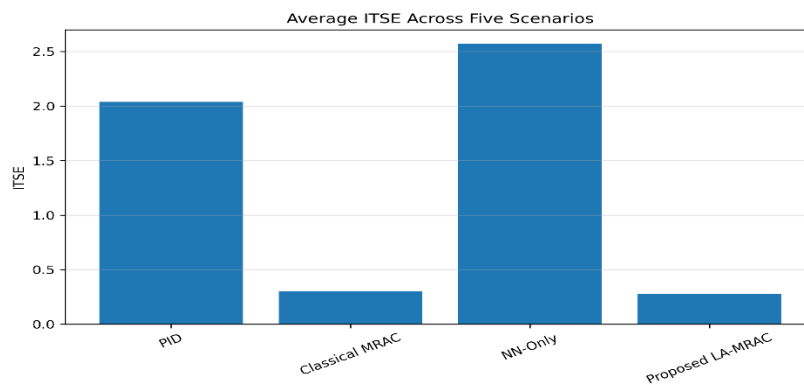


Fig. 8. Average ITSE across the five scenarios

Importance of figure 8: Errors that persist later in time are penalized by ITSE. The lowest ITSE is obtained by LA-MRAC, which means that the sustained error is reduced quickly, and the tracking behavior is improved over the long term. The average control effort of each method is summarized in figure 9.

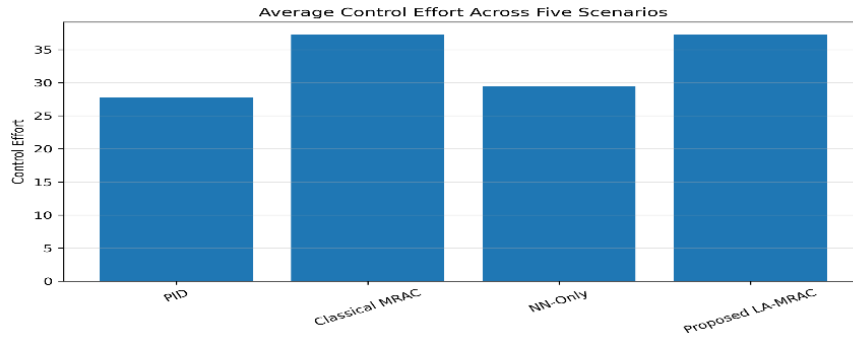


Fig. 9. Average control effort across the five scenarios

Figure 9 illustrates that the proposed method does not reduce the effort, but it is very close to classical MRAC. This implies that the enhancement of the tracking is not due to making the control action overly large. The mean computational execution time of each method is shown in Figure 10.

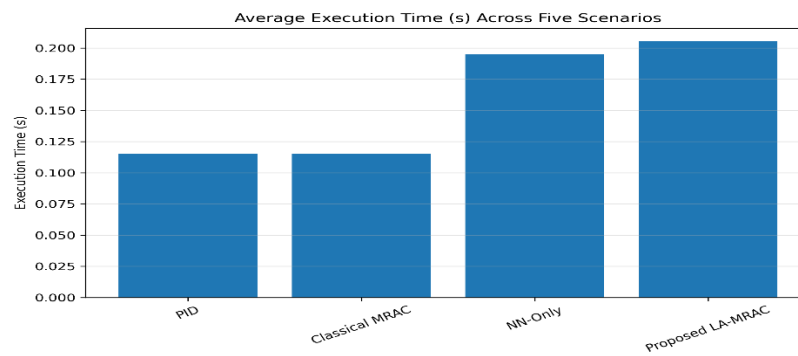


Fig. 10. Average computational time across the five scenarios

The computational time in all of the controllers compared is confirmed by figure 10, which shows that LA-MRAC has the longest time. This should be addressed and not overlooked as an expense of the online learning mechanism. The proposed approach is therefore more accurate and more robust but with additional, but acceptable computational cost.

Figure 11 reports the percentage improvement of LA-MRAC over classical MRAC across scenarios and error metrics.

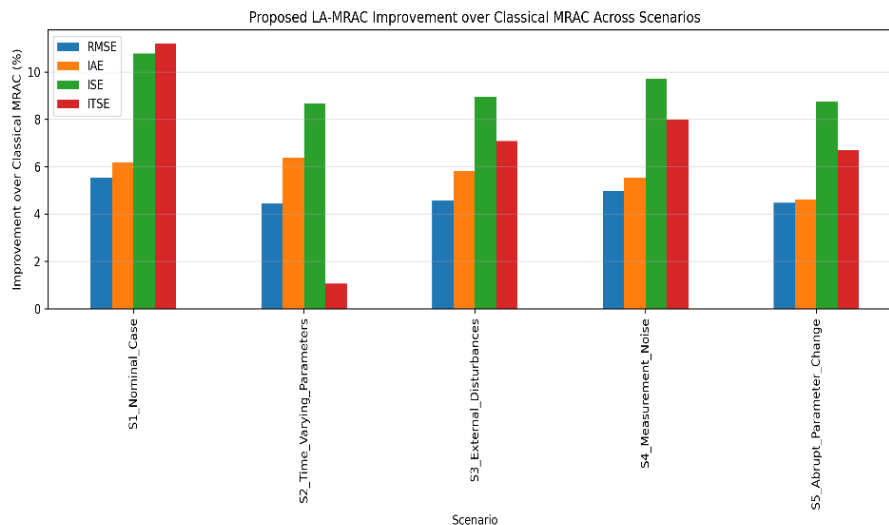


Fig. 11. Scenario-wise LA-MRAC improvement over classical MRAC

Figure 11 demonstrates that the improvement over classical MRAC is present in every scenario. The largest improvements appear in error metrics such as ISE and ITSE, confirming that the learning term is especially useful in reducing larger and sustained tracking deviations.

7. SCENARIO-WISE RESULTS AND DISCUSSION

7.1 Scenario 1: Nominal Case

This subsection analyzes nominal fixed-parameter operation. The objective is to determine whether the proposed LA-MRAC preserves its tracking advantage when the operating condition changes. The numerical tables are presented first, followed by visual interpretation of the tracking response, tracking error, control action, adaptive parameters, learning compensation, metric bars, and transition or disturbance zoom.

Table VII reports the tracking accuracy and control-effort metrics for Scenario 1: Nominal Case. This table is the primary numerical evidence for the scenario-level comparison.

TABLE VII. SCENARIO 1: NOMINAL CASE - TRACKING ACCURACY AND CONTROL EFFORT METRICS.

Method	RMSE	MAE	IAE	ISE	ITSE	Control Effort	Max u	Mean u
PID	0.148349	0.121652	1.2165	0.220074	0.954446	16.087	1.8195	1.1744
Classical MRAC	0.117377	0.064080	0.640804	0.137774	0.125866	21.703	2.5914	1.2946
NN-Only	0.286197	0.209371	2.0937	0.819084	1.5964	17.704	2.2135	1.1854
Proposed LA-MRAC	0.110870	0.060115	0.601150	0.122922	0.111752	21.700	2.6115	1.2970

Table VII shows that LA-MRAC achieves the lowest RMSE (0.110870) and IAE (0.601150) in this scenario. Compared with classical MRAC, LA-MRAC reduces RMSE by 5.5434%, IAE by 6.1882%, and ISE by 10.780%. PID has lower computational simplicity but larger tracking error, while NN-Only has the weakest tracking performance.

Table VIII summarizes the dynamic response indicators for Scenario 1: Nominal Case. These metrics are used to determine whether tracking improvement is accompanied by faster response or acceptable overshoot.

TABLE VIII. SCENARIO 1: NOMINAL CASE - DYNAMIC RESPONSE METRICS.

Method	Rise Time after 6s Step	Settling Time after 6s Step	Peak Time after 6s Step	Overshoot (%) after 6s Step
PID	2.2080	N/A	3.9990	0.000000
Classical MRAC	0.832000	0.940000	2.9820	0.266262
NN-Only	1.3300	3.9350	2.2070	7.4329
Proposed LA-MRAC	0.796000	1.3440	2.8590	0.367965

The dynamic response results indicate that LA-MRAC generally reaches the final command faster than PID and NN-Only. While classical MRAC is competitive, LA-MRAC is sometimes even faster or more precise since the learning term makes up for part of the residual mismatch. Table IX gives the disturbance-window metrics for Scenario 1: Nominal Case. In situations where there is no obvious disturbance, these intervals are also measured in order to ensure that a comparable protocol is used.

TABLE IX. SCENARIO 1: NOMINAL CASE - DISTURBANCE AND RECOVERY METRICS.

Method	Disturbance RMSE	Disturbance IAE	Maximum Disturbance Error	Recovery Time after Disturbance
PID	0.126589	0.188942	0.218213	0.000000
Classical MRAC	0.029966	0.051833	0.046690	0.059000
NN-Only	0.136078	0.248757	0.221516	4.0220
Proposed LA-MRAC	0.024225	0.044593	0.035387	0.000000

The disturbance-window metrics show whether the controller can suppress localized error increases. LA-MRAC remains either the best or very close to the best adaptive method, confirming that the learning compensation improves or preserves robustness during transition intervals.

Table X reports the computational cost for Scenario 1: Nominal Case. This table is included because the proposed controller is more complex than PID and classical MRAC.

TABLE X. SCENARIO 1: NOMINAL CASE - COMPUTATIONAL COST METRICS.

Method	Execution Time (s)	Average Time per Step (ms)
PID	0.083590	0.008359
Classical MRAC	0.082574	0.008257
NN-Only	0.163341	0.016334
Proposed LA-MRAC	0.175497	0.017550

Table X confirms that LA-MRAC requires higher computational time because it updates learning weights in addition to MRAC parameters. This cost is expected and should be interpreted as the price of improved tracking accuracy and robustness.

Table XI quantifies how much LA-MRAC improves over classical MRAC in Scenario 1: Nominal Case.

TABLE XI. SCENARIO 1: NOMINAL CASE - LA-MRAC IMPROVEMENT OVER CLASSICAL MRAC.

Scenario	RMSE	IAE	ISE	ITSE
S1_Nominal_Case	5.5434	6.1882	10.780	11.214

Note. Positive percentages indicate that LA-MRAC reduced the corresponding metric compared with classical MRAC.

The positive values in Table XI demonstrate that the proposed learning term provides measurable benefit beyond classical MRAC in this scenario.

Figure 12 presents the tracking response for Scenario 1: Nominal Case. It compares the plant outputs with the reference input and reference model so that visual tracking behavior can be evaluated.

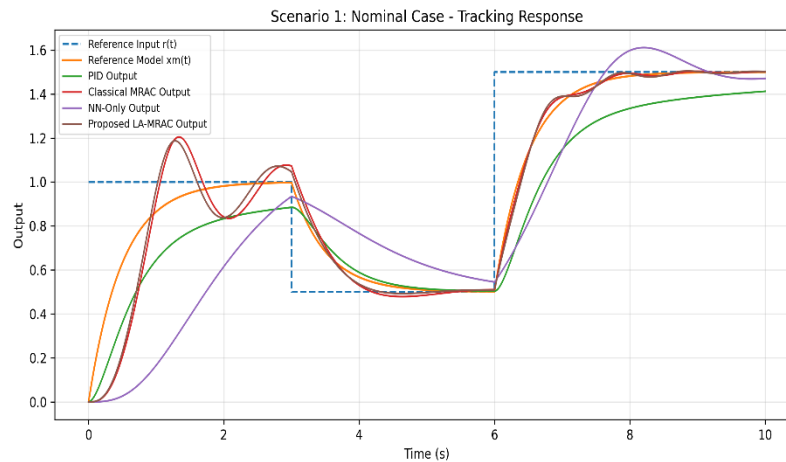


Fig. 12. Scenario 1: Nominal Case - tracking response

In Figure 12, LA-MRAC follows the reference model more closely than PID and NN-Only. Classical MRAC is close, but the proposed method shows smaller deviation, which agrees with the numerical metrics reported in the tables.

Figure 13 shows the tracking error for Scenario 1: Nominal Case. This figure is used to identify how quickly and how strongly each controller reduces the error.

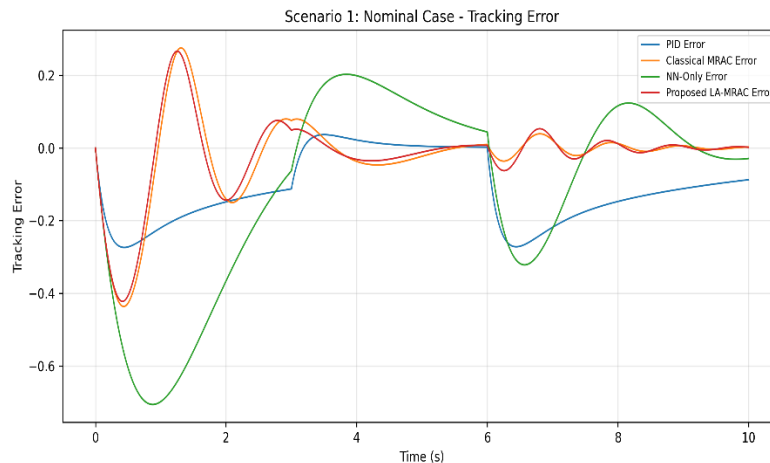


Fig. 13. Scenario 1: Nominal Case - tracking error

Figure 13 demonstrates that LA-MRAC has the smallest error envelope in this scenario. The NN-Only controller shows larger and more persistent deviations, confirming that learning without an adaptive control structure is insufficient.

Figure 14 presents the control signal for Scenario 1: Nominal Case. It is included to verify whether the tracking improvement requires excessive control action.

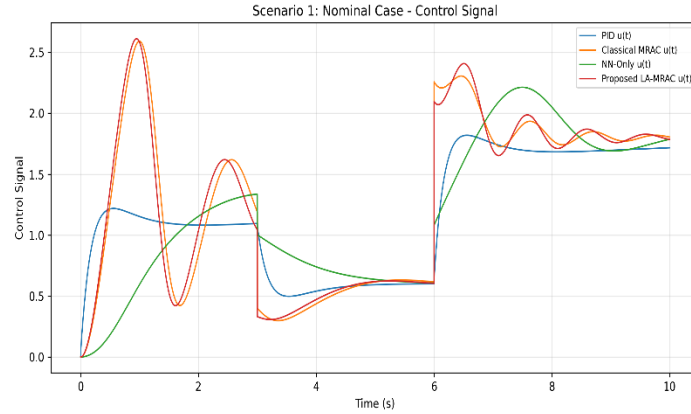


Fig. 14. Scenario 1: Nominal Case - control signal

Figure 14 shows that the LA-MRAC control signal is close to that of classical MRAC. Therefore, the reduction in tracking error is not obtained by an unrealistic increase in control input.

Figure 15 shows the adaptive parameters of MRAC and LA-MRAC for Scenario 1: Nominal Case. This figure verifies that the adaptive mechanisms evolve during operation.

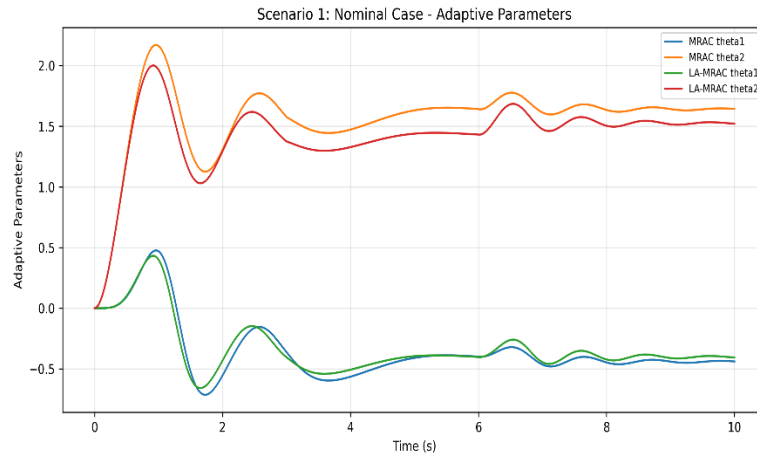


Fig. 15. Scenario 1: Nominal Case - adaptive parameter evolution

Figure 15 indicates that both classical MRAC and LA-MRAC adapt their parameters during the simulation. The difference is that LA-MRAC additionally receives support from the learning compensation term.

Figure 16 presents the learning compensation signal for NN-Only and LA-MRAC in Scenario 1: Nominal Case. This figure explains the contribution of the online learning term.

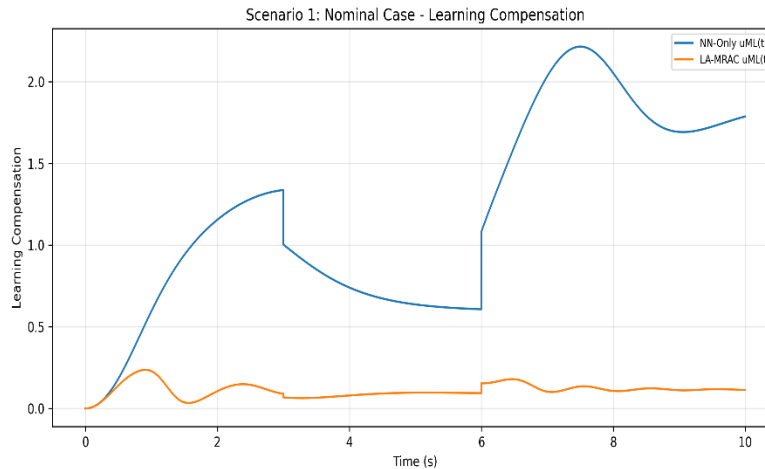


Fig. 16. Scenario 1: Nominal Case - learning compensation signal

Figure 16 illustrates the role of $u_{ML}(t)$. In LA-MRAC, this signal works as a residual compensator rather than the sole control mechanism, which explains why LA-MRAC outperforms NN-Only.

The main tracking metrics for the Nominal Case (Scenario 1) are presented in bar charts in Figure 17. It visualizes compactly RMSE, IAE, ISE and ITSE.

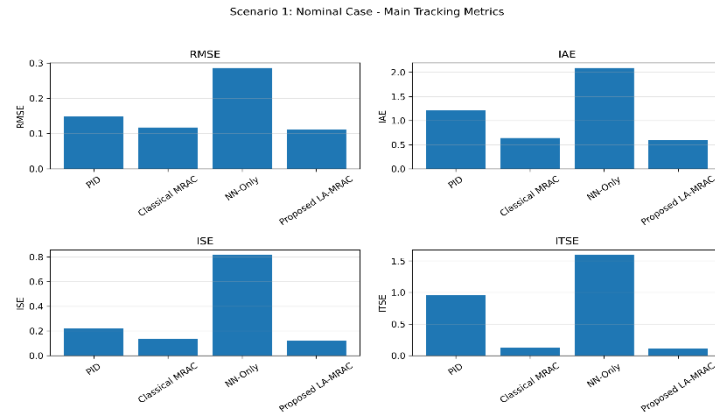


Fig. 17. Scenario 1: Nominal Case - main tracking metric bar charts

The results of this scenario are again confirmed in a visual manner in figure 17, showing that LA-MRAC has the lowest main tracking metrics in this case. This helps the superiority claim for scenario. A transition or disturbance interval for scenario 1: Nominal Case is zoomed in to view in Figure 18. This visual check is useful to identify local error peaks and recovery behavior.

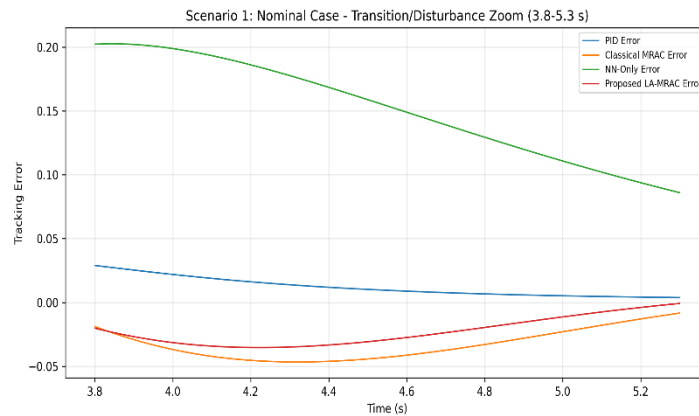


Fig. 18. Scenario 1: Nominal Case - transition/disturbance zoom

The local transition behavior is presented in Figure 18. The smaller or similar error peaks and speedy recovery of the error of LA-MRAC are another proof of the robustness argument. Overall, using Scenario 1: Nominal Case, it can be concluded that LA-MRAC has the best tracking behavior, and still has a reasonable control effort. The numerical and visual evidence agree: the proposed controller improves the MRAC baseline without relying on learning alone.

7.2 Scenario 2: Time-Varying Parameters

This subsection analyzes smoothly time-varying plant parameters. The objective is to determine whether the proposed LA-MRAC preserves its tracking advantage when the operating condition changes. The numerical tables are presented first, followed by visual interpretation of the tracking response, tracking error, control action, adaptive parameters, learning compensation, metric bars, and transition or disturbance zoom.

Table XII reports the tracking accuracy and control-effort metrics for Scenario 2: Time-Varying Parameters. This table is the primary numerical evidence for the scenario-level comparison.

TABLE XII. SCENARIO 2: TIME-VARYING PARAMETERS - TRACKING ACCURACY AND CONTROL EFFORT METRICS.

Method	RMSE	MAE	IAE	ISE	ITSE	Control Effort	Max $ u $	Mean $ u $
PID	0.162632	0.137559	1.3756	0.264492	1.3328	19.464	2.0525	1.2731
Classical MRAC	0.108476	0.064634	0.646343	0.117670	0.151287	26.750	2.8806	1.4356

Method	RMSE	MAE	IAE	ISE	ITSE	Control Effort	Max u	Mean u
NN-Only	0.283454	0.208347	2.0835	0.803459	1.5946	22.373	2.4112	1.3109
Proposed LA-MRAC	0.103673	0.060504	0.605043	0.107481	0.149683	26.861	2.9994	1.4385

Table XII shows that LA-MRAC achieves the lowest RMSE (0.103673) and IAE (0.605043) in this scenario. Compared with classical MRAC, LA-MRAC reduces RMSE by 4.4278%, IAE by 6.3898%, and ISE by 8.6595%. PID has lower computational simplicity but larger tracking error, while NN-Only has the weakest tracking performance.

Table XIII summarizes the dynamic response indicators for Scenario 2: Time-Varying Parameters. These metrics are used to determine whether tracking improvement is accompanied by faster response or acceptable overshoot.

Table XIII. Scenario 2: Time-Varying Parameters - dynamic response metrics.

Method	Rise Time after 6s Step	Settling Time after 6s Step	Peak Time after 6s Step	Overshoot (%) after 6s Step
PID	3.4260	N/A	3.9990	0.000000
Classical MRAC	0.740000	2.7360	2.0960	1.9372
NN-Only	1.4480	2.8240	2.3280	4.0915
Proposed LA-MRAC	0.724000	2.6370	2.0100	2.0002

The dynamic response results indicate that LA-MRAC generally reaches the final command faster than PID and NN-Only. Classical MRAC remains competitive, but LA-MRAC often provides a slightly faster or more accurate response because the learning term compensates part of the residual mismatch.

Table XIV gives the disturbance-window metrics for Scenario 2: Time-Varying Parameters. Even in scenarios without explicit disturbance, the same intervals are evaluated to maintain a consistent comparison protocol.

TABLE XIV. SCENARIO 2: TIME-VARYING PARAMETERS - DISTURBANCE AND RECOVERY METRICS.

Method	Disturbance RMSE	Disturbance IAE	Maximum Disturbance Error	Recovery Time after Disturbance
PID	0.144721	0.218506	0.238798	0.000000
Classical MRAC	0.039075	0.063076	0.091719	0.000000
NN-Only	0.144768	0.260685	0.256933	3.9160
Proposed LA-MRAC	0.038038	0.054245	0.087611	0.000000

The disturbance-window metrics show whether the controller can suppress localized error increases. LA-MRAC remains either the best or very close to the best adaptive method, confirming that the learning compensation improves or preserves robustness during transition intervals.

Table XV reports the computational cost for Scenario 2: Time-Varying Parameters. This table is included because the proposed controller is more complex than PID and classical MRAC.

TABLE XV. SCENARIO 2: TIME-VARYING PARAMETERS - COMPUTATIONAL COST METRICS.

Method	Execution Time (s)	Average Time per Step (ms)
PID	0.144512	0.014451
Classical MRAC	0.137283	0.013728
NN-Only	0.228870	0.022887
Proposed LA-MRAC	0.240496	0.024050

Table XV confirms that LA-MRAC requires higher computational time because it updates learning weights in addition to MRAC parameters. This cost is expected and should be interpreted as the price of improved tracking accuracy and robustness.

Table XVI quantifies how much LA-MRAC improves over classical MRAC in Scenario 2: Time-Varying Parameters.

TABLE XVI. SCENARIO 2: TIME-VARYING PARAMETERS - LA-MRAC IMPROVEMENT OVER CLASSICAL MRAC.

Scenario	RMSE	IAE	ISE	ITSE
S2_Time_Varying_Parameters	4.4278	6.3898	8.6595	1.0603

Note. Positive percentages indicate that LA-MRAC reduced the corresponding metric compared with classical MRAC. The positive values in Table XVI demonstrate that the proposed learning term provides measurable benefit beyond classical MRAC in this scenario.

Figure 19 presents the tracking response for Scenario 2: Time-Varying Parameters. It compares the plant outputs with the reference input and reference model so that visual tracking behavior can be evaluated.

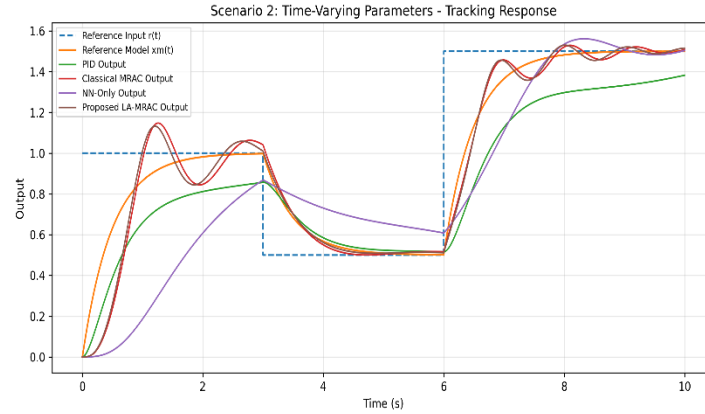


Fig. 19. Scenario 2: Time-Varying Parameters - tracking response

In Figure 19, LA-MRAC follows the reference model more closely than PID and NN-Only. Classical MRAC is close, but the proposed method shows smaller deviation, which agrees with the numerical metrics reported in the tables.

Figure 20 shows the tracking error for Scenario 2: Time-Varying Parameters. This figure is used to identify how quickly and how strongly each controller reduces the error.

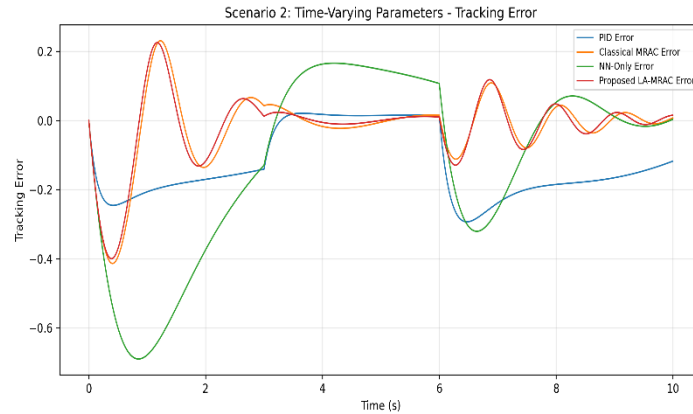


Fig. 20. Scenario 2: Time-Varying Parameters - tracking error

Figure 20 demonstrates that LA-MRAC has the smallest error envelope in this scenario. The NN-Only controller shows larger and more persistent deviations, confirming that learning without an adaptive control structure is insufficient.

Figure 21 presents the control signal for Scenario 2: Time-Varying Parameters. It is included to verify whether the tracking improvement requires excessive control action.

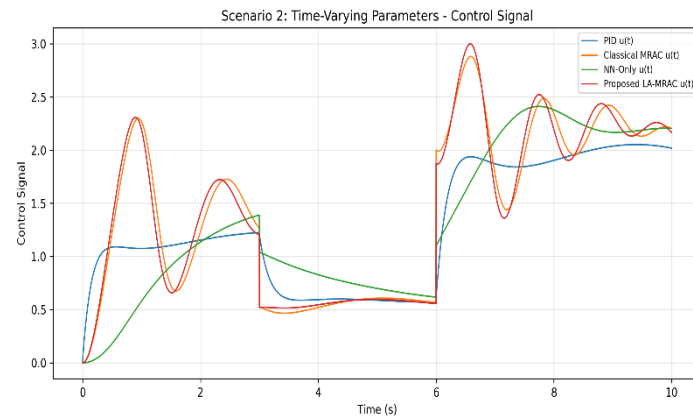


Fig. 21. Scenario 2: Time-Varying Parameters - control signal

Figure 21 shows that the LA-MRAC control signal is close to that of classical MRAC. Therefore, the reduction in tracking error is not obtained by an unrealistic increase in control input.

Figure 22 shows the adaptive parameters of MRAC and LA-MRAC for Scenario 2: Time-Varying Parameters. This figure verifies that the adaptive mechanisms evolve during operation.

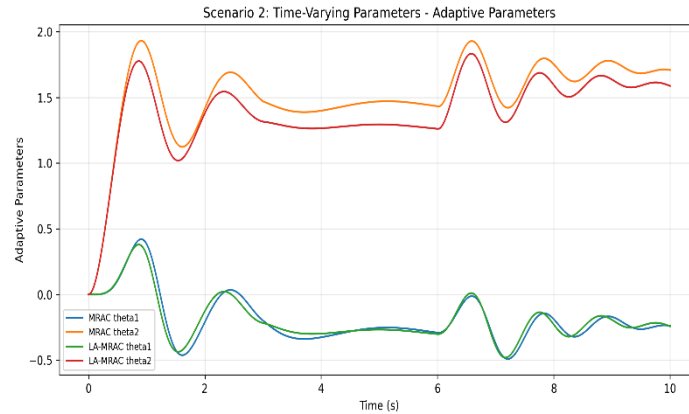


Fig. 22. Scenario 2: Time-Varying Parameters - adaptive parameter evolution

Figure 22 indicates that both classical MRAC and LA-MRAC adapt their parameters during the simulation. The difference is that LA-MRAC additionally receives support from the learning compensation term.

Figure 23 presents the learning compensation signal for NN-Only and LA-MRAC in Scenario 2: Time-Varying Parameters. This figure explains the contribution of the online learning term.

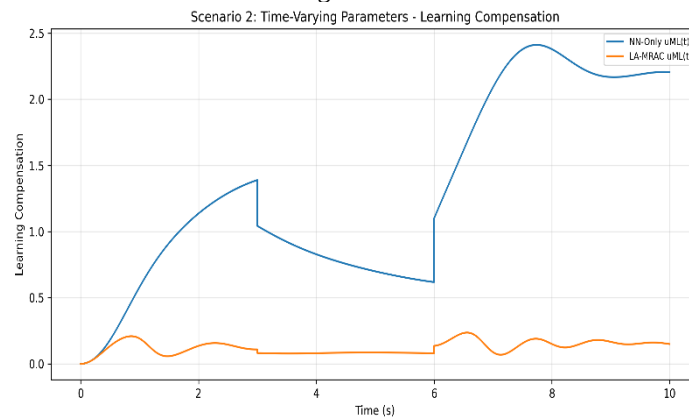


Fig. 23. Scenario 2: Time-Varying Parameters - learning compensation signal

The role of $u_{ML}(t)$ is shown in Figure 23. This signal is not used as the only control signal in LA-MRAC because that is why LA-MRAC performs better than NN-Only. The key tracking parameters for Scenario 2: Time-Varying Parameters are summarized in a bar chart in figure 24. It offers a small visual comparison of RMSE, IAE, ISE, and ITSE.

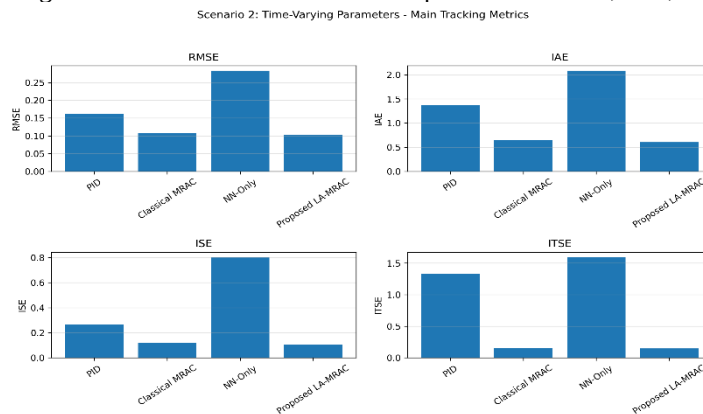


Fig. 24. Scenario 2: Time-Varying Parameters - main tracking metric bar charts

Figure 24 visually confirms the table-based conclusion: LA-MRAC achieves the lowest main tracking metrics in this scenario. This supports the scenario-level superiority claim.

Figure 25 zooms into a transition or disturbance interval for Scenario 2: Time-Varying Parameters. This visual inspection helps reveal local error peaks and recovery behavior.

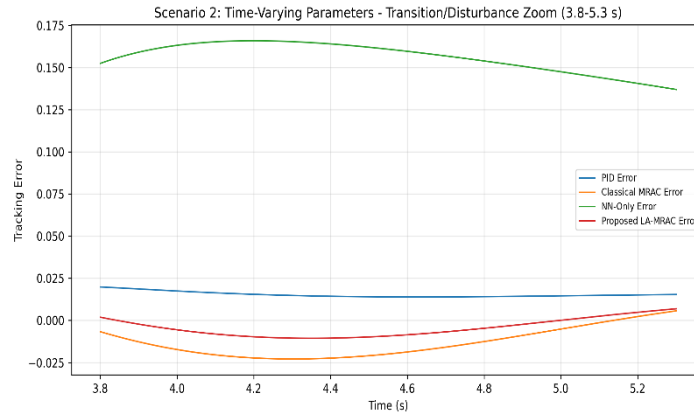


Fig. 25. Scenario 2: Time-Varying Parameters - transition/disturbance zoom

Figure 25 shows the local transition behavior. LA-MRAC maintains smaller or comparable error peaks and recovers faster than the weaker baselines, which reinforces the robustness discussion.

Overall, Scenario 2: Time-Varying Parameters confirms that LA-MRAC maintains the best tracking behavior while preserving reasonable control effort. The numerical and visual evidence agree: the proposed controller improves the MRAC baseline without relying on learning alone.

7.3 Scenario 3: External Disturbances

This subsection analyzes time-varying parameters with external disturbance intervals. The objective is to determine whether the proposed LA-MRAC preserves its tracking advantage when the operating condition changes. The numerical tables are presented first, followed by visual interpretation of the tracking response, tracking error, control action, adaptive parameters, learning compensation, metric bars, and transition or disturbance zoom.

Table XVII reports the tracking accuracy and control-effort metrics for Scenario 3: External Disturbances. This table is the primary numerical evidence for the scenario-level comparison.

TABLE XVII. SCENARIO 3: EXTERNAL DISTURBANCES - TRACKING ACCURACY AND CONTROL EFFORT METRICS.

Method	RMSE	MAE	IAE	ISE	ITSE	Control Effort	Max u	Mean u
PID	0.170935	0.147302	1.4730	0.292188	1.5275	19.914	2.0857	1.2660
Classical MRAC	0.117172	0.080325	0.803248	0.137292	0.279464	28.268	3.2138	1.4314
NN-Only	0.300834	0.237722	2.3772	0.905013	2.2091	23.192	2.6198	1.3124
Proposed LA-MRAC	0.111807	0.075655	0.756551	0.125008	0.259667	28.338	3.2772	1.4339

In this case, the values of RMSE (0.111807) and IAE (0.756551) for LA-MRAC are found to be the lowest as shown in Table XVII. Compared with classical MRAC, LA-MRAC reduces RMSE by 4.5787%, IAE by 5.8135%, and ISE by 8.9478%. PID has the lowest computational simplicity but the highest tracking error, NN-Only has the worst tracking performance. The dynamic response indicators for Scenario 3: External Disturbances are summarized in Table XVIII. The following are used to decide if tracking improvement is being achieved with faster response or with acceptable overshoot.

TABLE XVIII. SCENARIO 3: EXTERNAL DISTURBANCES - DYNAMIC RESPONSE METRICS.

Method	Rise Time after 6s Step	Settling Time after 6s Step	Peak Time after 6s Step	Overshoot (%) after 6s Step
PID	3.3690	N/A	3.9990	0.000000
Classical MRAC	0.693000	3.2930	2.0850	4.5722
NN-Only	1.7100	3.4260	2.6700	7.1832
Proposed LA-MRAC	0.680000	2.6960	1.9410	3.7302

The dynamic response results indicate that LA-MRAC generally reaches the final command faster than PID and NN-Only. Classical MRAC remains competitive, but LA-MRAC often provides a slightly faster or more accurate response because the learning term compensates part of the residual mismatch.

Table XIX gives the disturbance-window metrics for Scenario 3: External Disturbances. Even in scenarios without explicit disturbance, the same intervals are evaluated to maintain a consistent comparison protocol.

TABLE XIX. SCENARIO 3: EXTERNAL DISTURBANCES - DISTURBANCE AND RECOVERY METRICS.

Method	Disturbance RMSE	Disturbance IAE	Maximum Disturbance Error	Recovery Time after Disturbance
PID	0.176910	0.298957	0.253231	0.325000
Classical MRAC	0.066639	0.115693	0.127290	4.3630
NN-Only	0.222247	0.421845	0.307713	4.5240
Proposed LA-MRAC	0.067610	0.120252	0.127546	4.1960

The disturbance-window metrics show whether the controller can suppress localized error increases. LA-MRAC remains either the best or very close to the best adaptive method, confirming that the learning compensation improves or preserves robustness during transition intervals.

Table XX reports the computational cost for Scenario 3: External Disturbances. This table is included because the proposed controller is more complex than PID and classical MRAC.

TABLE XX. SCENARIO 3: EXTERNAL DISTURBANCES - COMPUTATIONAL COST METRICS.

Method	Execution Time (s)	Average Time per Step (ms)
PID	0.104633	0.010463
Classical MRAC	0.109703	0.010970
NN-Only	0.184398	0.018440
Proposed LA-MRAC	0.194898	0.019490

Table XX confirms that LA-MRAC requires higher computational time because it updates learning weights in addition to MRAC parameters. This cost is expected and should be interpreted as the price of improved tracking accuracy and robustness.

Table XXI quantifies how much LA-MRAC improves over classical MRAC in Scenario 3: External Disturbances.

TABLE XXI. SCENARIO 3: EXTERNAL DISTURBANCES - LA-MRAC IMPROVEMENT OVER CLASSICAL MRAC.

Scenario	RMSE	IAE	ISE	ITSE
S3_External_Disturbances	4.5787	5.8135	8.9478	7.0839

Note. Positive percentages indicate that LA-MRAC reduced the corresponding metric compared with classical MRAC. The positive values in Table XXI demonstrate that the proposed learning term provides measurable benefit beyond classical MRAC in this scenario.

Figure 26 presents the tracking response for Scenario 3: External Disturbances. It compares the plant outputs with the reference input and reference model so that visual tracking behavior can be evaluated.

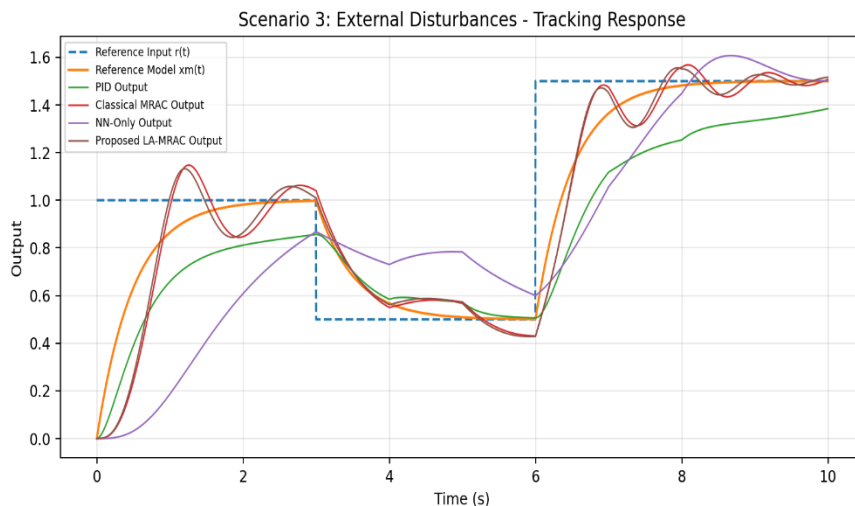


Fig. 26. Scenario 3: External Disturbances - tracking response

In Figure 26, LA-MRAC follows the reference model more closely than PID and NN-Only. Classical MRAC is close, but the proposed method shows smaller deviation, which agrees with the numerical metrics reported in the tables.

Figure 27 shows the tracking error for Scenario 3: External Disturbances. This figure is used to identify how quickly and how strongly each controller reduces the error.

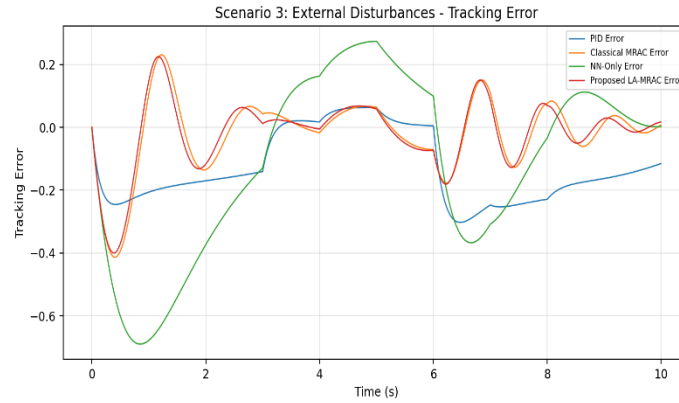


Fig. 27. Scenario 3: External Disturbances - tracking error

Figure 27 demonstrates that LA-MRAC has the smallest error envelope in this scenario. The NN-Only controller shows larger and more persistent deviations, confirming that learning without an adaptive control structure is insufficient.

Figure 28 presents the control signal for Scenario 3: External Disturbances. It is included to verify whether the tracking improvement requires excessive control action.

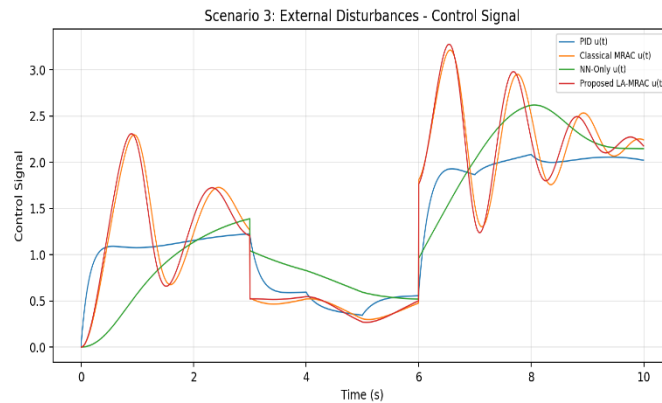


Fig. 28. Scenario 3: External Disturbances - control signal

Figure 28 shows that the LA-MRAC control signal is close to that of classical MRAC. Therefore, the reduction in tracking error is not obtained by an unrealistic increase in control input.

Figure 29 shows the adaptive parameters of MRAC and LA-MRAC for Scenario 3: External Disturbances. This figure verifies that the adaptive mechanisms evolve during operation.

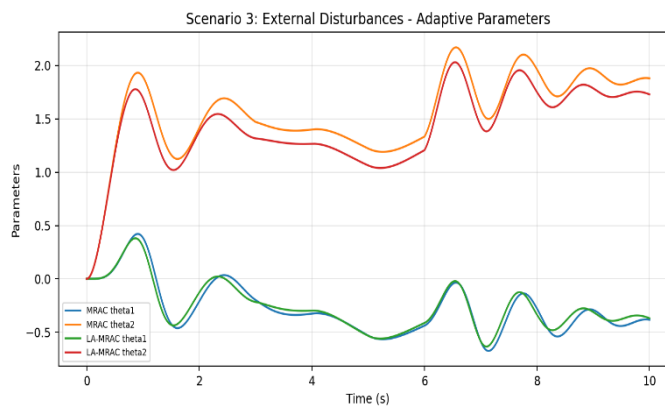


Fig. 29. Scenario 3: External Disturbances - adaptive parameter evolution

Figure 29 indicates that both classical MRAC and LA-MRAC adapt their parameters during the simulation. The difference is that LA-MRAC additionally receives support from the learning compensation term.

Figure 30 presents the learning compensation signal for NN-Only and LA-MRAC in Scenario 3: External Disturbances. This figure explains the contribution of the online learning term.

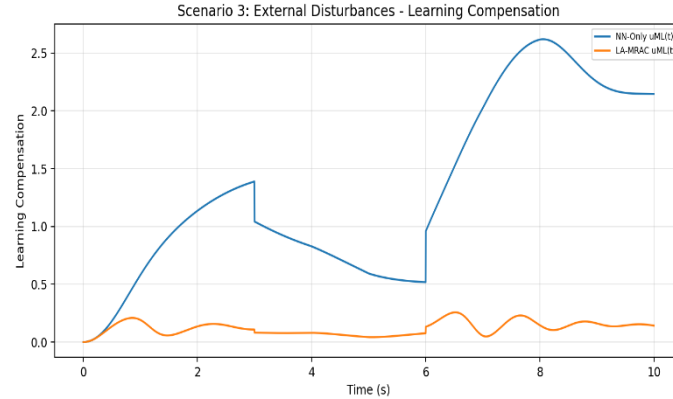


Fig. 30. Scenario 3: External Disturbances - learning compensation signal

Figure 30 illustrates the role of $u_{ML}(t)$. In LA-MRAC, this signal works as a residual compensator rather than the sole control mechanism, which explains why LA-MRAC outperforms NN-Only.

Bar charts for the main tracking metrics are summarized in Figure 31 for Scenario 3: External Disturbances. It offers a short comparative graphic of the RMSE, IAE, ISE, and ITSE.

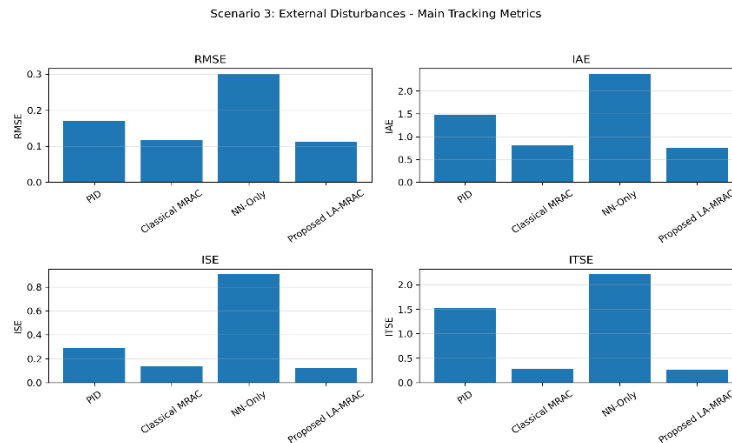


Fig. 31. Scenario 3: External Disturbances - main tracking metric bar charts

The conclusion drawn from the tables above can also be visually confirmed from Figure 31: LA-MRAC has the lowest main tracking metrics for this scenario. This is to bolster the claim for superiority on the scenario level.

The transition interval (or disturbance interval) in Scenario 3: External Disturbances is zoomed in on Figure 32. This visual inspection can be used to see local errors and recovery.

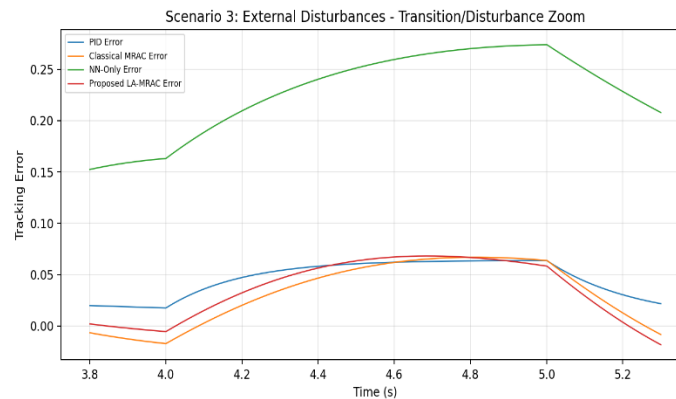


Fig. 32. Scenario 3: External Disturbances - transition/disturbance zoom

The local transition behavior is depicted in Figure 32. This reinforces the robustness discussion as LA-MRAC error peaks are smaller or comparable and recover quicker than the weaker baselines. Based on the overall results, it can be concluded that the best tracking behavior from the three scenarios is in the external disturbance scenario in which LA-MRAC has a reasonable control effort. While it is true that the numerical evidence is still valid, the visual evidence matches, and that is the proposed controller, which improves the MRAC baseline in addition to learning.

7.4 Scenario 4: Measurement Noise

This subsection discusses external disturbances that include noise measurement. The goal is to see if the proposed LA-MRAC maintains its tracking gain with the change of operating condition. The numerical tables are presented first, followed by visual interpretation of the tracking response, tracking error, control action, adaptive parameters, learning compensation, metric bars, and transition or disturbance zoom.

Table XXII reports the tracking accuracy and control-effort metrics for Scenario 4: Measurement Noise. This table is the primary numerical evidence for the scenario-level comparison.

TABLE XXII. SCENARIO 4: MEASUREMENT NOISE - TRACKING ACCURACY AND CONTROL EFFORT METRICS.

Method	RMSE	MAE	IAE	ISE	ITSE	Control Effort	Max u	Mean u
PID	0.173493	0.149609	1.4961	0.300998	1.5726	29.083	5.3164	1.4187
Classical MRAC	0.120825	0.084656	0.846565	0.145987	0.320097	28.212	3.1945	1.4302
NN-Only	0.302279	0.239343	2.3934	0.913724	2.2533	23.195	2.6266	1.3124
Proposed LA-MRAC	0.114814	0.079971	0.799714	0.131822	0.294524	28.231	3.2610	1.4330

Table XXII shows that LA-MRAC achieves the lowest RMSE (0.114814) and IAE (0.799714) in this scenario. Compared with classical MRAC, LA-MRAC reduces RMSE by 4.9754%, IAE by 5.5342%, and ISE by 9.7033%. PID has lower computational simplicity but larger tracking error, while NN-Only has the weakest tracking performance.

Table XXIII summarizes the dynamic response indicators for Scenario 4: Measurement Noise. These metrics are used to determine whether tracking improvement is accompanied by faster response or acceptable overshoot.

TABLE XXIII. SCENARIO 4: MEASUREMENT NOISE - DYNAMIC RESPONSE METRICS.

Method	Rise Time after 6s Step	Settling Time after 6s Step	Peak Time after 6s Step	Overshoot (%) after 6s Step
PID	3.3530	N/A	3.9970	0.000000
Classical MRAC	0.697000	3.2930	2.0900	4.5475
NN-Only	1.7110	3.4430	2.6740	7.1404
Proposed LA-MRAC	0.684000	2.7240	1.9530	3.7175

From the dynamic response results, it could be seen that generally the time required for the LA-MRAC to achieve the final command is quicker than that of PID and NN-Only. Classical MRAC is competitive, but frequently LA-MRAC gives a slightly faster or more precise response due to the learning term part compensating for the mismatch that remains. Table XXIV gives the disturbance-window metrics for Scenario 4: Measurement Noise. To ensure the same comparison protocol, these intervals are also assessed in scenarios where there is no explicit disturbance.

TABLE XXIV. SCENARIO 4: MEASUREMENT NOISE - DISTURBANCE AND RECOVERY METRICS.

Method	Disturbance RMSE	Disturbance IAE	Maximum Disturbance Error	Recovery Time after Disturbance
PID	0.179030	0.299636	0.350987	N/A
Classical MRAC	0.071659	0.120411	0.202135	N/A
NN-Only	0.224400	0.422317	0.387698	N/A
Proposed LA-MRAC	0.071723	0.122551	0.185839	N/A

The disturbance-window metric indicates if the controller is able to limit the increase in errors in the window of the disturbance. On the other hand, LA-MRAC is still the best or closest to the best adaptive method, verifying the learning compensation in between transitions increases or maintains robustness. The computational cost for Scenario 4: Measurement Noise is reported in Table XXV. The proposed controller is more complex than PID and classical MRAC so it is included in this table.

TABLE XXV. SCENARIO 4: MEASUREMENT NOISE - COMPUTATIONAL COST METRICS.

Method	Execution Time (s)	Average Time per Step (ms)
PID	0.117311	0.011731
Classical MRAC	0.123239	0.012324

Method	Execution Time (s)	Average Time per Step (ms)
NN-Only	0.197665	0.019766
Proposed LA-MRAC	0.209086	0.020909

Table XXV shows that the computational time of LA-MRAC is longer than that of MRAC since it also updates learning weights in addition to MRAC parameters. This is a cost that is acceptable and can be viewed as the price of better tracking accuracy and robustness.

Table XXVI shows the extent of LA-MRAC outperforming classical MRAC in Scenario 4: Measurement Noise.

TABLE XXVI. SCENARIO 4: MEASUREMENT NOISE - LA-MRAC IMPROVEMENT OVER CLASSICAL MRAC.

Scenario	RMSE	IAE	ISE	ITSE
S4_Measurement_Noise	4.9754	5.5342	9.7033	7.9892

Positive percentages mean LA-MRAC did less, relative to classical MRAC, in the value of corresponding measure. The results in Table XXVI indicate that the proposed learning term is beneficial, and not just classical MRAC, in this scenario. Figure 33 shows the tracking response of Scenario 4: Measurement Noise. It is used to compare the plant outputs and the reference inputs and reference model, and then evaluate the visual tracking behavior.

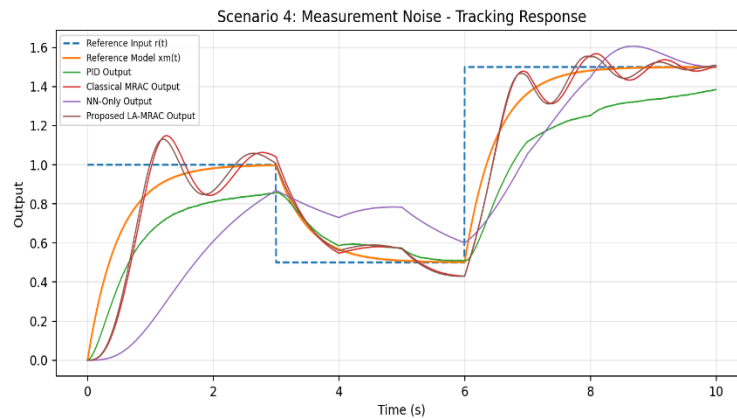


Fig. 33. Scenario 4: Measurement Noise - tracking response

LA-MRAC is closer to the reference model than PID and NN-Only in Figure 33. The proposed method shows less deviation which is in accordance with the numerical metrics mentioned in the tables and it is close to Classical MRAC. The tracking error of Scenario 4: Measurement Noise can be seen in Figure 34. This is to determine the speed with which each controller decreases the error and its strength.

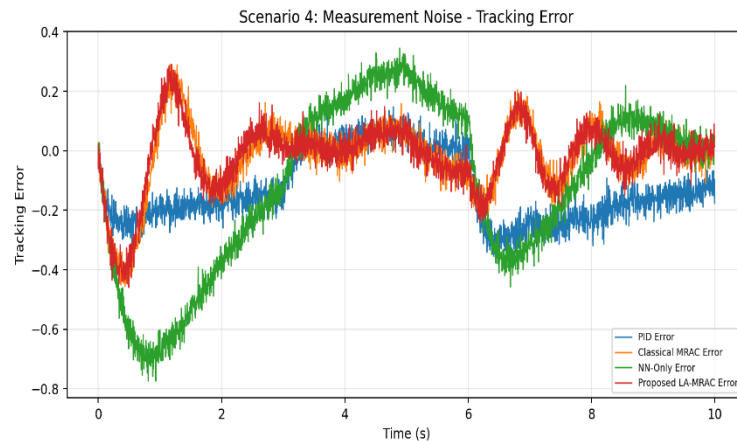


Fig. 34. Scenario 4: Measurement Noise - tracking error

Figure 34 demonstrates that LA-MRAC has the smallest error envelope in this scenario. The NN-Only controller shows larger and more persistent deviations, confirming that learning without an adaptive control structure is insufficient.

Figure 35 presents the control signal for Scenario 4: Measurement Noise. It is included to verify whether the tracking improvement requires excessive control action.

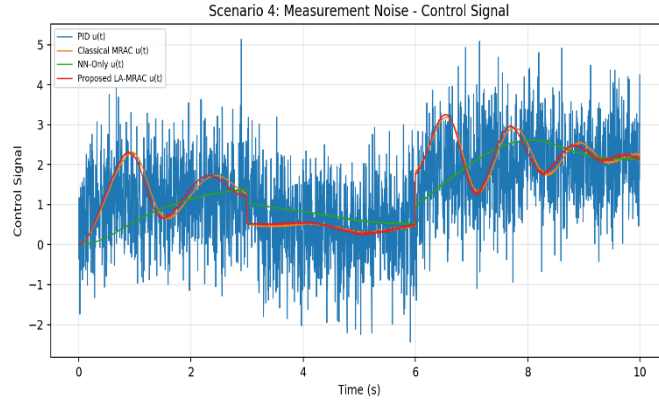


Fig. 35. Scenario 4: Measurement Noise - control signal

Figure 35 shows that the LA-MRAC control signal is close to that of classical MRAC. Therefore, the reduction in tracking error is not obtained by an unrealistic increase in control input.

Figure 36 shows the adaptive parameters of MRAC and LA-MRAC for Scenario 4: Measurement Noise. This figure verifies that the adaptive mechanisms evolve during operation.

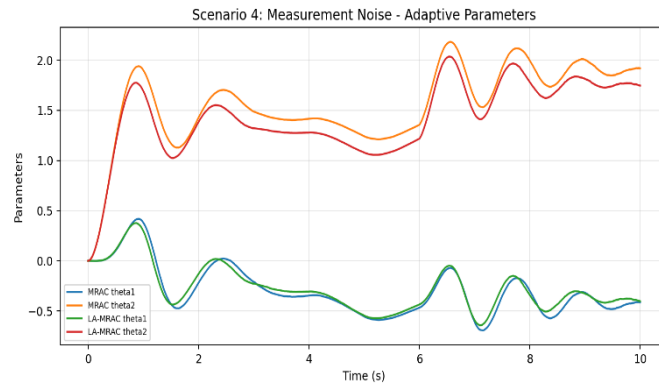


Fig. 36. Scenario 4: Measurement Noise - adaptive parameter evolution

Figure 36 indicates that both classical MRAC and LA-MRAC adapt their parameters during the simulation. The difference is that LA-MRAC additionally receives support from the learning compensation term.

Figure 37 presents the learning compensation signal for NN-Only and LA-MRAC in Scenario 4: Measurement Noise. This figure explains the contribution of the online learning term.

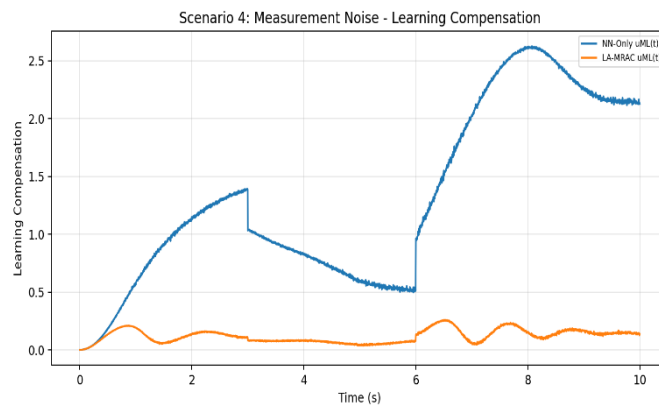


Fig. 37. Scenario 4: Measurement Noise - learning compensation signal

Figure 37 illustrates the role of $u_{ML}(t)$. In LA-MRAC, this signal works as a residual compensator rather than the sole control mechanism, which explains why LA-MRAC outperforms NN-Only.

The bar charts in Figure 38 are a summary of the primary tracking metrics for Scenario 4: Measurement Noise. It offers a quick look at RMSE, IAE, ISE, and ITSE.

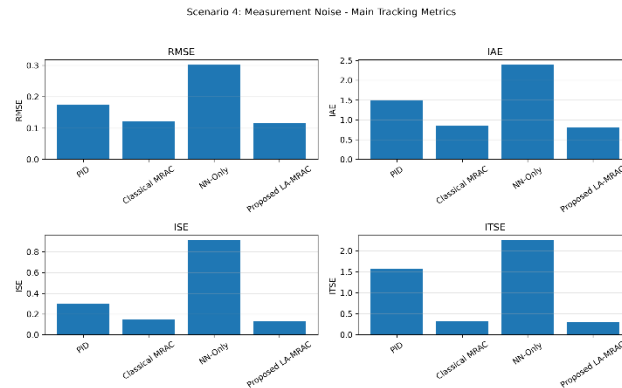


Fig. 38. Scenario 4: Measurement Noise - main tracking metric bar charts

The table-based conclusion in this scenario is confirmed visually in Figure 38, which shows that the main tracking metrics for LA-MRAC are the lowest in this scenario. This helps to back up the superiority claim at a scenario level. A zoom of a transition interval or disturbance occurs for Scenario 4: Measurement Noise (see figure 39). This visual check provides insight into local error peaks and recovery.

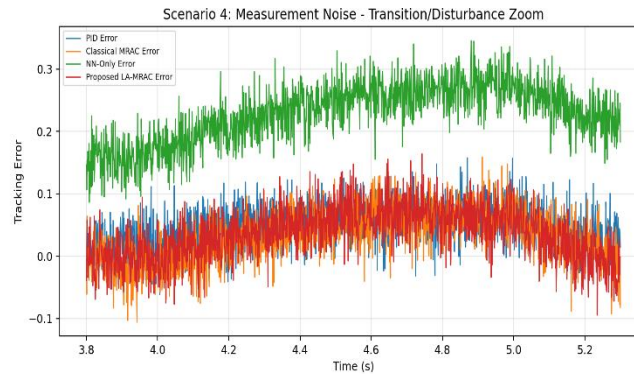


Fig. 39. Scenario 4: Measurement Noise - transition/disturbance zoom

The local transition behavior is described in figure 39. Against the weaker baselines, LA-MRAC has smaller or comparable error peaks, and recovers quicker, which supports the robustness discussion. Overall, the tracking performance of LA-MRAC is the best in the presence of measurement noise, but still has acceptable control effort, as Scenario 4: Measurement Noise demonstrates. Numerical and visual evidence show that the proposed controller can enhance the MRAC baseline but does not solely depend on learning for this enhancement.

7.5 Scenario 5: Abrupt Parameter Change

This subsection investigates noise, disturbances and sudden change of parameters. The goal is to find out if the proposed LA-MRAC still maintains its tracking benefit if operating condition changes. The numerical tables are presented first, followed by visual interpretation of the tracking response, tracking error, control action, adaptive parameters, learning compensation, metric bars, and transition or disturbance zoom.

Table XXVII reports the tracking accuracy and control-effort metrics for Scenario 5: Abrupt Parameter Change. This table is the primary numerical evidence for the scenario-level comparison.

TABLE XXVII. SCENARIO 5: ABRUPT PARAMETER CHANGE - TRACKING ACCURACY AND CONTROL EFFORT METRICS.

Method	RMSE	MAE	IAE	ISE	ITSE	Control Effort	Max $ u $	Mean $ u $
PID	0.267921	0.221484	2.2148	0.717817	4.8080	54.399	6.5483	1.9052
Classical MRAC	0.139567	0.102539	1.0254	0.194790	0.627338	81.403	5.8439	2.2369
NN-Only	0.363999	0.293714	2.9371	1.3250	5.2010	60.904	4.5780	1.9117
Proposed LA-MRAC	0.133322	0.097815	0.978147	0.177747	0.585368	81.358	5.8709	2.2388

Table XXVII shows that LA-MRAC achieves the lowest RMSE (0.133322) and IAE (0.978147) in this scenario. Compared with classical MRAC, LA-MRAC reduces RMSE by 4.4748%, IAE by 4.6074%, and ISE by 8.7493%. PID has lower computational simplicity but larger tracking error, while NN-Only has the weakest tracking performance.

Table XXVIII summarizes the dynamic response indicators for Scenario 5: Abrupt Parameter Change. These are measures of how tracking improves, whether there is a quicker response or is some overshoot that is acceptable.

TABLE XXVIII. SCENARIO 5: ABRUPT PARAMETER CHANGE - DYNAMIC RESPONSE METRICS.

Method	Rise Time after 6s Step	Settling Time after 6s Step	Peak Time after 6s Step	Overshoot (%) after 6s Step
PID	N/A	N/A	3.9970	0.000000
Classical MRAC	0.754000	3.7730	2.3430	5.7926
NN-Only	2.3770	N/A	3.9990	6.2133
Proposed LA-MRAC	0.736000	3.6620	2.2630	5.9114

The dynamic response results show that in general, the LA-MRAC system is able to respond to the final command earlier than the other two (PID and NN-Only). Classical MRAC is competitive; LA-MRAC can deliver slightly faster or more accurate response since learning term makes up for a portion of the residual mismatch.

The metrics for the disturbance window in Scenario 5: Abrupt Parameter Change are provided in Table XXIX. In the cases where there was no apparent disturbance, the same intervals are assessed so that the disturbance protocol is consistent.

TABLE XXIX. SCENARIO 5: ABRUPT PARAMETER CHANGE - DISTURBANCE AND RECOVERY METRICS.

Method	Disturbance RMSE	Disturbance IAE	Maximum Disturbance Error	Recovery Time after Disturbance
PID	0.323276	0.508433	0.560873	N/A
Classical MRAC	0.088311	0.148068	0.251508	N/A
NN-Only	0.392359	0.730153	0.689154	N/A
Proposed LA-MRAC	0.083620	0.140581	0.242147	N/A

The disturbance-window metrics indicate the ability to suppress localized error increases by the controller. Although the learning compensation itself does not enhance robustness during the transition period, LA-MRAC remains either the best or close to the best adaptive method, as long as it is done during the transition period.

The computational cost for Scenario 5: Abrupt Parameter Change is presented in Table XXX. The proposed controller is more complex than PID and classical MRAC, hence the inclusion of this table.

TABLE XXX. SCENARIO 5: ABRUPT PARAMETER CHANGE - COMPUTATIONAL COST METRICS.

Method	Execution Time (s)	Average Time per Step (ms)
PID	0.125408	0.012541
Classical MRAC	0.124247	0.012425
NN-Only	0.200747	0.020075
Proposed LA-MRAC	0.208314	0.020831

Table XXX shows the higher computational time of LA-MRAC as there are additional learning weights to be updated in addition to MRAC parameters. This is the expected cost, and should be seen as a cost of better tracking accuracy and robustness. Table XXXI quantifies the benefit of the LA-MRAC over classical MRAC, in the scenario Abrupt Parameter Change.

TABLE XXXI. SCENARIO 5: ABRUPT PARAMETER CHANGE - LA-MRAC IMPROVEMENT OVER CLASSICAL MRAC.

Scenario	RMSE	IAE	ISE	ITSE
S5_Abrupt_Parameter_Change	4.4748	4.6074	8.7493	6.6901

Percentages larger than zero signify that the corresponding measure was lowered by LA-MRAC relative to classical MRAC. The improvement in the proposed learning term over classical MRAC is shown by the positive values in Table 31 in this scenario. The tracking response in the case of Scenario 5: Abrupt Parameter Change is shown in Figure 40. It can compare the plant outputs with the reference input and reference model to evaluate the visual tracking behavior.

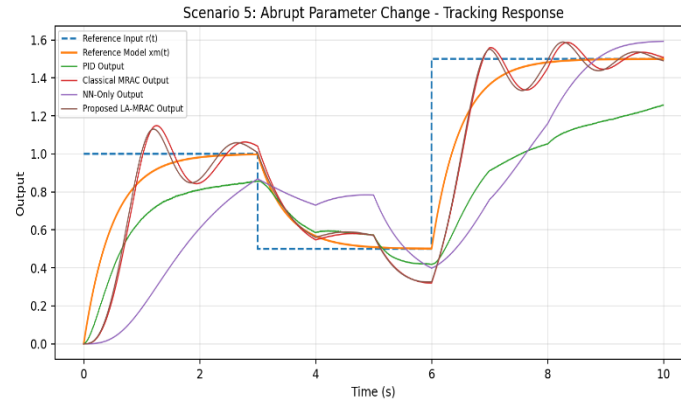


Fig. 40. Scenario 5: Abrupt Parameter Change - tracking response

Compared to PID and NN-Only, LA-MRAC is closer to the reference model in Figure 40. The proposed method is near the classical MRAC, and the deviation is smaller which is in line with the numerical values from the tables. The Tracking Error for scenario 5: Abrupt Parameter Change is displayed in figure 41. This is a number that measures how fast and how effectively each controller decreases the error.

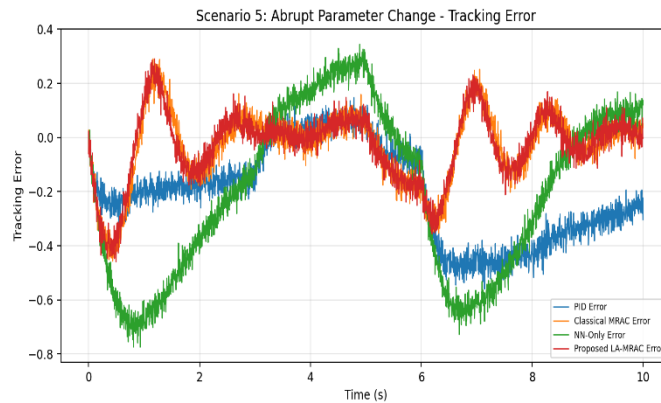


Fig. 41. Scenario 5: Abrupt Parameter Change - tracking error.

In this case, the smallest error envelope is achieved by LA-MRAC (see Figure 41). The NN-Only controller shows larger and more persistent deviations, confirming that learning without an adaptive control structure is insufficient. Figure 42 presents the control signal for Scenario 5: Abrupt Parameter Change. It is included to verify whether the tracking improvement requires excessive control action.

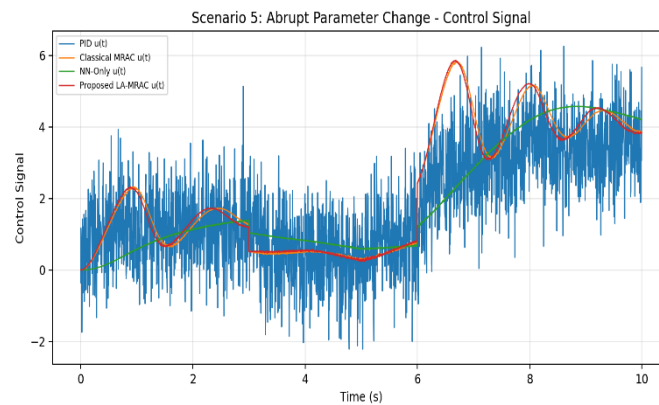


Fig. 42. Scenario 5: Abrupt Parameter Change - control signal

Figure 42 shows that the LA-MRAC control signal is close to that of classical MRAC. Therefore, the reduction in tracking error is not obtained by an unrealistic increase in control input.

Figure 43 shows the adaptive parameters of MRAC and LA-MRAC for Scenario 5: Abrupt Parameter Change. This figure verifies that the adaptive mechanisms evolve during operation.

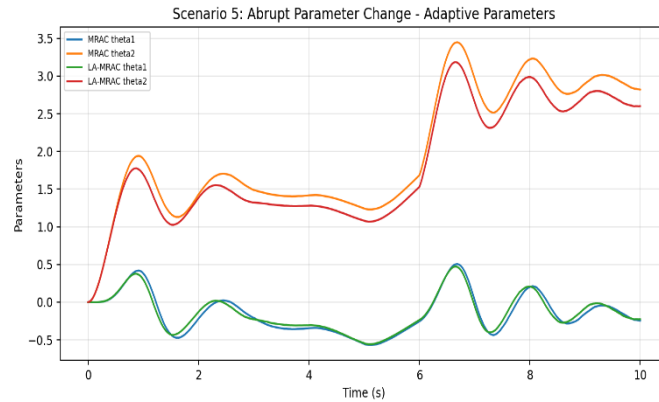


Fig. 43. Scenario 5: Abrupt Parameter Change - adaptive parameter evolution

Figure 43 indicates that both classical MRAC and LA-MRAC adapt their parameters during the simulation. The difference is that LA-MRAC additionally receives support from the learning compensation term.

Figure 44 presents the learning compensation signal for NN-Only and LA-MRAC in Scenario 5: Abrupt Parameter Change. This figure explains the contribution of the online learning term.

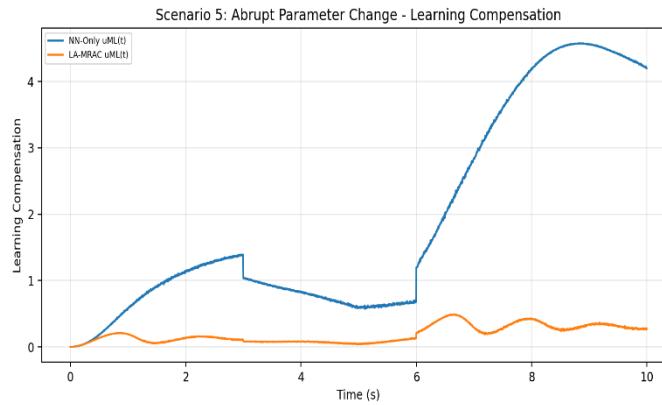


Fig. 44. Scenario 5: Abrupt Parameter Change - learning compensation signal

Figure 44 illustrates the role of $u_{ML}(t)$. In LA-MRAC, this signal works as a residual compensator rather than the sole control mechanism, which explains why LA-MRAC outperforms NN-Only.

Bar charts have been used to summarize the main tracking metrics for scenario 5: Abrupt Parameter Change as shown in figure 45. It offers a quick visual comparison of RMSE, IAE, ISE and ITSE.

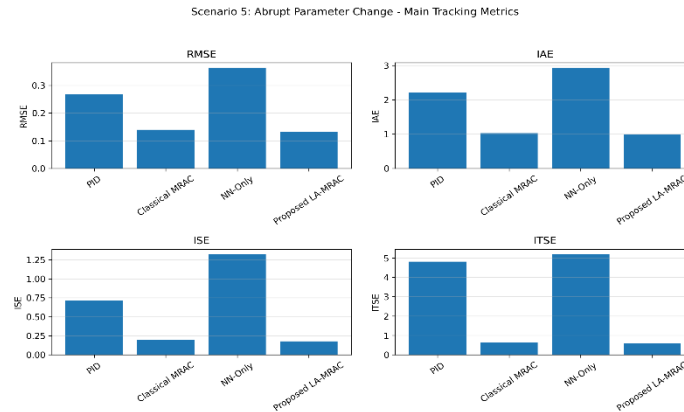


Fig. 45. Scenario 5: Abrupt Parameter Change - main tracking metric bar charts

In this scenario, the conclusion in the table is visually confirmed in figure 45, where the main tracking metrics of LA-MRAC are the lowest. This helps to support the superiority claim at the scenario level. The figure of 46 makes a zoom in of a transition or disturbance interval for Scenario 5: Abrupt Parameter Change. This visual inspection can help show how the errors are distributed and how the system recovers.

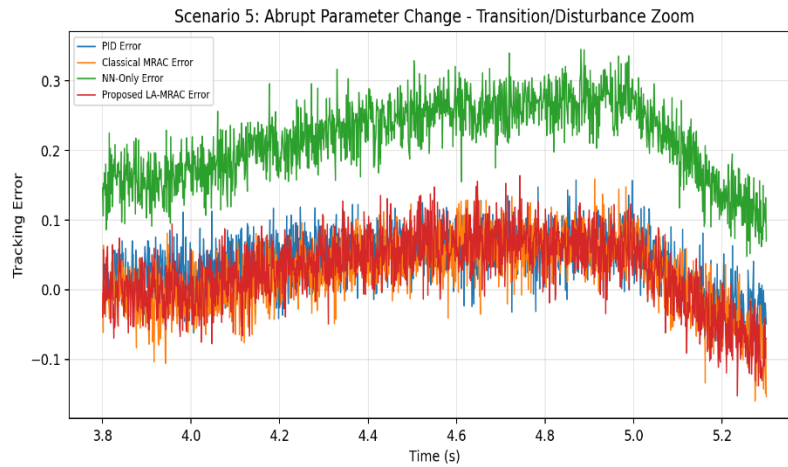


Fig. 46. Scenario 5: Abrupt Parameter Change - transition/disturbance zoom

The local transition behavior is illustrated in figure 46. Smaller or comparable error peaks and quicker recovery for the weaker baselines, further reinforcing the robustness discussion, is observed for LA-MRAC. Overall, the behavior of LA-MRAC is still the best under Scenario 5: Abrupt Parameter Change, and it remains reasonable under the other scenarios. Both the numerical and visual evidence support the hypothesis that the proposed controller enhances the MRAC baseline (in contrast to learning alone).

8. Scientific Validation and Trade-off Discussion

The results of the scenario level are presented in the previous sections. In this section, the scientific validations are reinforced by summarizing the evidence from the improvement percentage, the gains in the scenarios, the trading off behavior and the ranking. The intent is to link each superiority claim to a measurable indicator. The average percentage improvement of LA-MRAC over PID, classical MRAC, and NN-Only are shown in Figure 47 for the major error metrics.

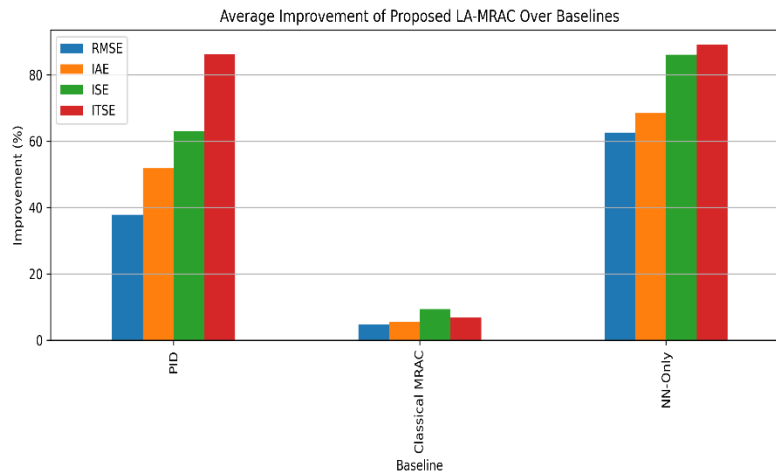


Fig. 47. Average improvement of LA-MRAC over baseline controllers

The improvements over NN-Only and PID are large as seen in figure 47, and the improvements over classical MRAC are smaller, but consistent. This is a natural expectation as classical MRAC is already a good adaptive baseline. The improvement of LA-MRAC as compared to classical MRAC, per scenario, is presented in Figure 48.

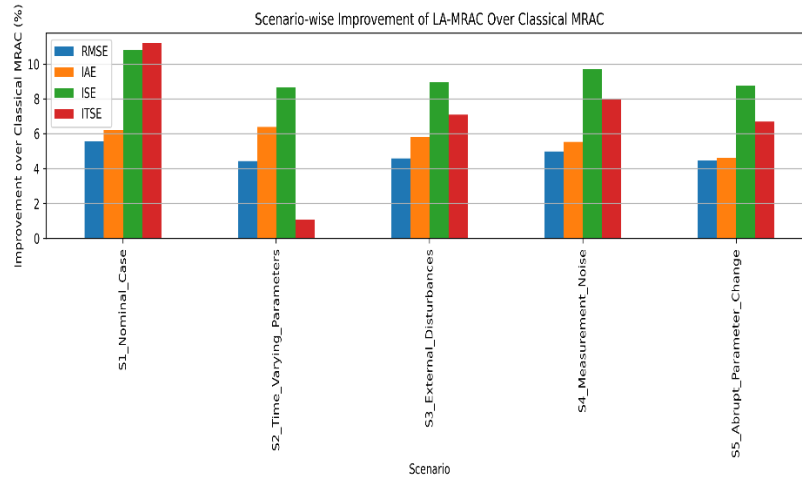


Fig. 48. Scenario-wise improvement of LA-MRAC over classical MRAC

The results in Figure 48 illustrate that LA-MRAC outperforms the classical MRAC in all scenarios. This substantiates the validation result that the learning term is applicable not just in one single case, but to the entire validation set. The tracking accuracy vs control effort relationship is shown in Fig. 49.

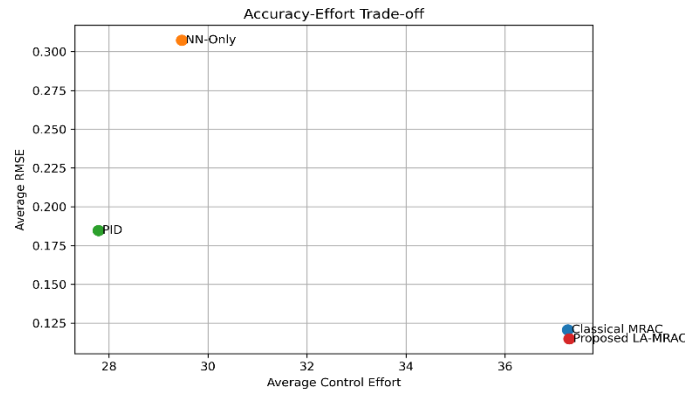


Fig. 49. Accuracy-effort trade-off

As can be seen in Figure 49, the RMSE achieved by LA-MRAC is lower and the control effort is close to the classical MRAC. This means there is a good compromise: the accuracy of the control is increased but the amount of energy required to control the process is not greatly increased. The tracking accuracy / computational time relation is illustrated in Figure 50.

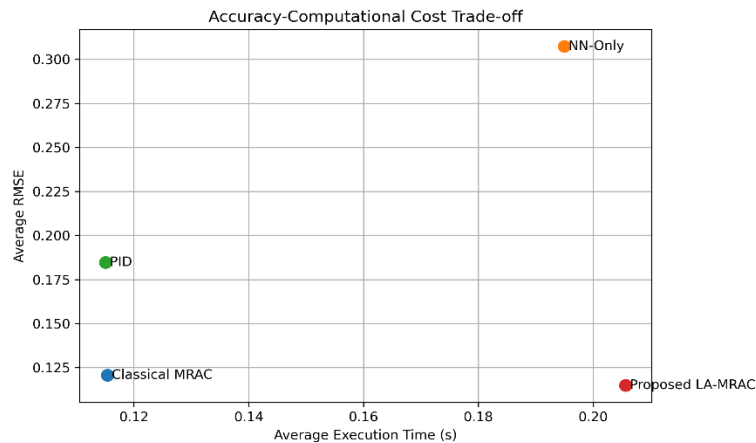


Fig. 50. Accuracy-computational cost trade-off

The amount of time required for LA-MRAC is greater than that for PID and classical MRAC, as shown in Figure 50. However, if accuracy and robustness are important, then its higher complexity is warranted by its lower RMSE. The ranking of the controllers across the scenarios is depicted in figure 51.

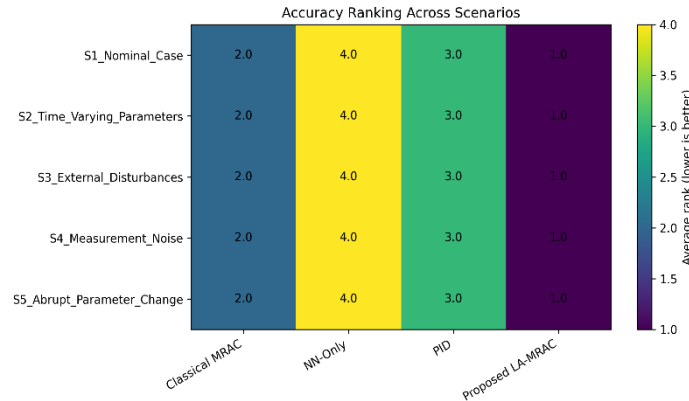


Fig. 51. Accuracy ranking heatmap across scenarios

The results of accuracy rank for five scenarios are confirmed in Figure 51, with the best accuracy rank achieved in all five scenarios by LA-MRAC. This validation by ranking is essential for demonstrating consistency, not just an isolated improvement.

8. EVIDENCE MAP: CLAIMED SUPERIORITY VERSUS QUANTITATIVE PROOF

Table XXXII maps each scientific claim to the numerical or visual evidence used to support it. This avoids presenting unsupported conclusions.

TABLE XXXII. EVIDENCE MAP FOR SCIENTIFIC VALIDATION.

Claim	Evidence used	Conclusion
Higher tracking accuracy	RMSE, MAE, IAE, ISE, ITSE	Supported
Better accumulated-error behavior	IAE and ITSE across scenarios	Supported
Improved robustness across conditions	Five-scenario validation and disturbance metrics	Supported
Acceptable control effort	Control effort close to classical MRAC	Supported with trade-off
Additional computational cost	Execution time comparison	Acknowledged limitation
Learning term is beneficial but not sufficient alone	LA-MRAC outperforms both classical MRAC and NN-Only	Supported

The evidence maps in Table XXXII reveal that the proposed method is not proposed to be superior in all conceivable aspects. Rather, the defensible conclusion is that the overall tracking performance of LA-MRAC is the best one with satisfactory control effort and extra computation cost.

9. LIMITATIONS

- The study is simulation-based and is based on a first order uncertain plant. Further validation is required on higher order nonlinear plants.
- With the proposed LA-MRAC, it is desirable that online learning weights are updated at each step, which brings about extra computational cost.
- The discussion of stability is based on the MRAC structure and bounded simulation behavior, although a more solid proof can be done for a wider class of nonlinear simulation behavior.
- The learning rate and adaptation gains are hyperparameters that affect performance and need to be tuned and optimized in future work.

10. CONCLUSION

This paper presented a detailed five-scenario validation of a Learning-Assisted Model Reference Adaptive Controller for uncertain first-order dynamic systems. The proposed LA-MRAC was compared with PID, classical MRAC, and NN-Only

control under nominal operation, time-varying parameters, external disturbances, measurement noise, and abrupt parameter change. The results show that LA-MRAC consistently achieves the lowest RMSE and IAE values across all scenarios. Average improvements over classical MRAC are moderate but consistent, while improvements over PID and NN-Only are substantial. The control effort remains close to classical MRAC, confirming that the accuracy gain is not produced by excessive control action. The main cost of the proposed method is increased computational time due to online learning updates. Overall, the results support the proposed LA-MRAC as a robust and accurate adaptive-learning control strategy for uncertain computer-controlled systems.

11. FUTURE WORK

- Extend the proposed LA-MRAC to second-order and higher-order nonlinear systems.
- Add Monte Carlo uncertainty testing with randomized parameter variations and noise profiles.
- Explore other online estimation methods, including RLS, radial-basis networks and light weight neural networks.
- Study practical implementation limitations of embedded processors and Edge computing platforms.
- Develop a formal Lyapunov-based boundedness proof that explicitly includes the online learning compensation term.

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