

Research Article

Behavior of Square Reinforced Concrete Columns Wrapped with Carbon Fiber Reinforced Polymer Sheets Under Static and Dynamic LoadingIbtehal Abdulmonem Al-Naqdi^{1, *}, Ali Mohammed Rashid Almashhadani^{2,}, Sara Gh. Hussein^{3,}¹ Senior Chief Engineer, Ministry of Education, Baghdad10001, Iraq.² Iraqi University, Baghdad10001, Iraq.³ Chief Engineer, Ministry of Education, Baghdad10001, Iraq.**ARTICLEINFO**

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**ABSTRACT**

This article provides a detailed numerical study of reinforced concrete columns wrapped with carbon fiber-reinforced polymer sheets (CFRP) using the Finite Element Method (FEM) by ANSYS APDL (Ansys Parametric Design Language). This study aims to investigate nonlinear structural behavior, including maximum displacements under maximum axial loading, maximum axial stress, and maximum axial plastic strain. Both static and harmonic analyses were conducted. A multilinear isotropic hardening model (MISO) is utilized for concrete and a bilinear isotropic hardening model for the steel rebars to model the material nonlinearity. Rebars beams elements join with concrete at the nodes between them. Axial force is applied at the top center of the column. Results reveal the extent of enhancement of elastic strain, stress, and axial load capacity of the column studied. The maximum load of the 4-sided wrapped column was 1.34 times that of the RC columns. The confinement pressure was greater in the concrete core of wrapped columns with CFRP sheets. During the application of harmonic loading, a considerable reduction in stresses was noticed due to wrapping. The results of this article emphasize that a more detailed FEM analysis of CFRP-wrapped RC columns is highly essential in the process of warranting performance-based design and retrofitting techniques.

1. INTRODUCTION

Reinforced concrete (RC) columns are important load-bearing elements in most structural systems. The stability of buildings around the world is established by how they perform under the influence of axial and lateral loads. With advanced age, however, or excessive requirements due to insufficient initial detailing, or seismic vulnerability, most old columns could not sustain the real structural requirements [1]. Carbon fiber-reinforced polymer (CFRP) composites are popular due to their strength-to-weight ratio, corrosion resistance, and ease of use, among others, as factors of external reinforcement techniques to enhance performance [2,3]. CFRP confinement significantly enhances the ultimate load-bearing capacity of RC columns in the axial direction by eliminating the lateral dilatation of the concrete core [3]. This enhances the compressive strength of confined concrete [4]. The experimental and numerical analysis shows that CFRP-wrapped columns can resist axial loads in the range of 100 percent of the unconfined ones, depending on the number of layers and wrap configuration [5,6]. CFRP wrapping enhances the ductility and energy absorption besides strength, which leads to an increase in the maximum displacement before failure [7]. This is necessary in seismic situations, where ultimate strength is less important than deformation capacity [8].

CFRP confinement delays cracking and failure, and this results in a more stable progressive stress-strain behavior. Material-wise, CFRP wrapping changes the distribution of stresses and decreases the stress concentrations in the concrete core, especially in the failure zone areas [9]. Besides, CFRP-confined concrete has a much higher elastic strain than unconfined columns, indicating a shift in the failure mode to ductile [10,11]. It is additionally proved by the finite element analysis (FEA) that the more homogeneous distribution of the plastic zone in CFRP-reinforced columns reduces local failures and enhances the overall structural stability [12].

The FEA can be effectively used to analyze the behavior of RC elements in which the members are wrapped with carbon fiber polymer sheets. Taking advantage of this, ANSYS has a large variety of element models [13, 14]. The effective

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analysis of the behavior of RC members wrapped with CFRP sheets is possible with the help of FEA. The maximum load increases greatly, and the final displacement reduces with an increase in the number of CFRP sheet layers [15, 16].

This article, based on nonlinear finite element modeling, investigates the influence of CFRP wrapping on RC columns by studying plastic strain distribution, maximum axial load, maximum displacement, and stress distribution. Through a modification of the properties of carbon fiber sheets during static and dynamic analysis, the results underscore the importance of proper confinement in enhancing structural column ductility and strength during axial compression scenarios [17, 18].

The novelty of the present study is that it combines experimental validation, an analytical benchmark from ACI PRC-440.2-23, a systematic static parametric study and a harmonic response analysis into a single finite element framework. Developed FE model was initially validated against the published experimental results and further benchmarked against ACI provisions, both experimentally and by code. The validated model was then used to explore the impact of the CFRP wrapping configuration, thickness and elastic modulus on the axial load capacity, displacement, stress and plastic strain of the reinforced concrete columns. Harmonic response analysis was also adopted to investigate the response of CFRP-strengthened columns to frequency-dependent dynamic excitations. This hybrid numerical approach allows to assess CFRP confinement in a more complete way as it brings together: static strength enhancement, parametric effects, experimental validation, code comparison, and dynamic response in the same study.

Although there are many experimental and numerical studies conducted on the reinforced concrete columns confined with CFRP, there are some drawbacks. Earlier studies have been on experimental behaviour, strengthening parameters, and only a few have developed a systematically validated FE model that can simulate the full load–displacement behaviour of CFRP-strengthened RC columns up to failure. Furthermore, direct comparisons of the finite element predictions with the experimental and existing design provisions, such as ACI PRC-440.2-23, are still limited. A detailed numerical study is then needed to assess the accuracy of FE models and to understand the effect of CFRP on the strength, stiffness, deformation behavior, and failure modes of RC columns.

2. ELEMENT TYPES

The 8-node 3-D SOLID65 element was used in ANSYS APDL to model the concrete. They have 3 translational degrees of freedom at each node (UX, UY, and UZ). Nonlinear modeling of concrete is widely carried out using SOLID65 since it has the ability to capture tensile cracking and compression crushing with the ANSYS concrete material formulation [14, 19]. The element is especially recommended for reinforced concrete structures where the concrete has different tensile/compressive characteristics. Cracking is modeled as smeared cracking instead of an explicit consideration of discrete cracks [20, 21].

The two-node BEAM188 element was used to model the longitudinal and transverse reinforcing bars. A three-dimensional beam element for linear and non-linear structural analysis, BEAM188. Each node possesses six degrees of freedom three translations (UX, UY and UZ) and three rotations (ROTX, ROTY and ROTZ) [22]. It thus may refer to the axial deformation, bending, torsion, and shear deformation of the reinforcing bars.

The eight-node SOLID45 element with three translational degrees of freedom at each node [21, 23] was used to model the loading and support plates made of steel. The CFRP sheets were modeled using the four-node SHELL181 element, which can be used for linear and nonlinear analysis of thin to moderately thick shell structures. There are six degrees of freedom (translation in three directions and rotation in three directions) at each node of SHELL181. The fiber orientation of the CFRP was designed to simulate the effect of circumferential confinement due to the external wrapping. The top of the column was equipped with a loading plate so that the stress concentration at a single concrete node is not unrealistic but transferred to the axial load [24-25].

The constitutive nonlinear behavior of concrete and reinforcing steel, as well as the isotropic behavior of the CFRP and the elastic behavior of steel loading plates, were explicitly modelled. The constitutive relationships adopted were chosen to capture the main mechanisms of the response of CFRP-strengthened reinforced concrete columns, concrete cracking and crushing, steel yielding, and the typically linear-elastic response of CFRP up to rupture.

TABLE I. MATERIAL MODEL SUMMARY

Material	Element	Constitutive model	Principal behavior represented
Concrete	SOLID65	Multilinear nonlinear concrete model with smeared cracking	Tensile cracking and compressive crushing
Reinforcing steel	BEAM188	Bilinear elastic-plastic	Yielding and post-yield response

Material	Element	Constitutive model	Principal behavior represented
CFRP	SHELL181	Isotropic linear-elastic	linear-elastic CFRP without explicit rupture modelling
Steel plates	SOLID45	Isotropic linear-elastic	Elastic load distribution

3. MATERIAL PROPERTIES

Properties values adopted during this study for defining materials are listed in (Tables II–VI). A bilinear model is used to model steel reinforcing rebars. The nonlinear isotropic hardening model (MISO) provided by the ANSYS APDL program is utilized to model nonlinear properties to concrete.

TABLE II. INPUT DATA FOR THE CONCRETE COLUMN.

Description of constants and material properties	Value
Density of concrete	2500 kg/m ³
Modulus of elasticity of concrete	34856 MPa
Poisson's ratio of concrete	0.2
Open shear transfer coef. for concrete	0.2
Closed shear transfer coef. for concrete	0.9
Uniaxial cracking stress for concrete,	3.39 MPa

TABLE III. INPUT DATA FOR THE STEEL REBARS.

Description of constants and material properties	Value
Density of steel	7850 kg/m ³
Modulus of elasticity of steel	200 GPa
Poisson's ratio of steel	0.3
Tensile strength*	413 MPa

TABLE IV. INPUT DATA FOR THE STEEL PLATE.

Description of constants and material properties	Value
Density of steel	7850 kg/m ³
Modulus of elasticity of steel	200 GPa
Poisson's ratio of steel	0.3

TABLE V. INPUT DATA FOR THE CFRP [16].

Description of constants and material properties	Value
Density of CFRP	1180 kg/m ³
Modulus of elasticity of CFRP	228 GPa
Poisson's ratio of CFRP	0.3
Tensile strength	4275 MPa
fiber orientation	45°

TABLE VI. INPUT DATA FOR EPOXY [16].

Description of constants and material properties	Value
Modulus of elasticity of Epoxy	30 GPa
Ultimate tensile strain	0.03

The resin used in the CFRP–concrete system is an epoxy adhesive 0.8 mm thick modeled as shell element, which serves as the bonding layer between the CFRP sheet and the concrete substrate. Its main function is to provide adhesion and facilitate efficient stress transfer from the concrete to the CFRP, thereby enabling composite action. In the numerical model, the

epoxy resin was assigned a modulus of elasticity of 30 GPa and a Poisson's ratio of 0.30, based on the material properties reported in [16].

4. MODELING METHODOLOGY

A mesh sensitivity study was conducted to understand the impact of the element size on the numerical results and also to select the optimum size of the mesh that is suitable for a good compromise between the accuracy of the solution and the computational efficiency. Four different sizes of mesh were used: 60, 50, 40 and 30 mm. The main convergence indicators were the ultimate axial load and the axial displacement. The difference between the ultimate axial load and displacement with the two meshes is 3.31% and 3.47%, respectively as seen in Table VII, when the mesh is refined from a 60 mm to a 50 mm. Further refinement from 50 mm to 40 mm did not significantly increase these differences to 2.02% and 2.70%, respectively, and further refinement from 40 mm to 30 mm only increased these differences to 1.20% and 1.33%, respectively. The results show that the solution tends to converge as the mesh becomes finer. The 30 mm and 40 mm meshes yielded slightly more accurate solutions, but the 50 mm mesh results were within about 3.20% and 4.00% errors for the ultimate axial load and displacement, respectively, from those of the finest 30 mm mesh. The differences were deemed to be acceptable for the purposes of the present study. So, in the following finite element analyses a mesh size of 50 mm was chosen for the concrete elements, taking both into account: numerical accuracy and computational efficiency. As explained earlier, it was realised that while the mesh was very fine and precise in modelling the structure, the resulting calculations would be more costly, time-consuming, and labour-intensive.

TABLE VII. MESH SENSITIVITY ANALYSIS BASED ON MAXIMUM AXIAL LOAD AND DISPLACEMENT

Mesh Size (mm)	Max. Axial Load (kN)	Max. Axial Displacement (mm)	Load Difference (%)	Displacement Difference (%)
60	1170	1.39	—	—
50	1210	1.44	3.31	3.47
40	1235	1.48	2.02	2.70
30	1250	1.50	1.20	1.33

Using the case of an RC column with idealized sharp-corner, with height of 3 meters and a cross-section area of (0.4*0.4) meters, the entire column was simulated as illustrated in (Figure 1.). The lower end was restrained in all degrees of freedom, and lateral displacement of the top end of the column was restrained. Steel plates were modelled at column ends, and the interaction faces between the end plates and the RC columns were modelled. In concrete and steel plate, meshing was permitted for each element with dimensions 50*50*50 mm. Rebars and ties were modelled as beam elements with given cross sections (major rebars had a diameter of 16 mm (steel ratio=0.01), and ties had a diameter of 10 mm). CFRP sheets are modelled as a shell with several layers each having a thickness of 0.125 mm, with a grid size of 50x50 mm. The CFRP was glued with 0.8 mm of epoxy between the CFRP and concrete. The ties were laid at a distance of $h/6$ at both ends, and at 250 mm centre-to-centre along the remaining length in order to minimize stresses. Contact surfaces are modelled between the concrete column and carbon fibre polymer sheets.

a)

b)

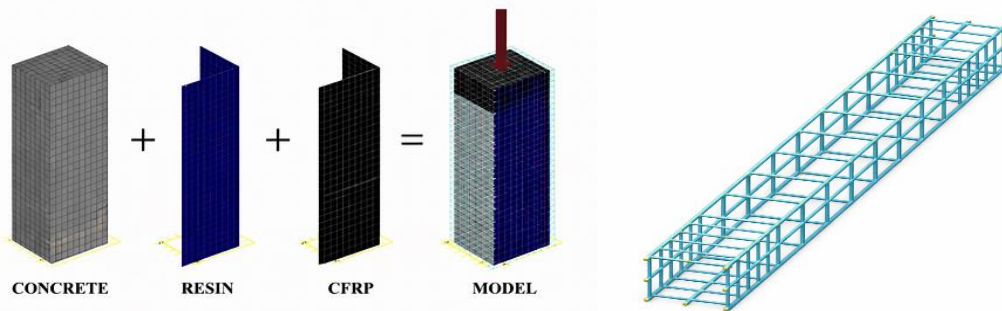


Fig. 1. Typical finite element model used in the analysis of confined concrete model loaded in compression- a), Model of column rebar- b)

5. VALIDATION OF THE FINITE ELEMENT MODEL

The developed finite element model was checked with the experimental studies reported by Rajai Z. Al-Rousan [16]. The experimental program involved an 750mm x 150mm circular column with reinforcement. The column selected for the experimental program for validation was a circular reinforced concrete column 750mm high and 150mm in diameter. Four 9.5 mm diameter steel bars were provided vertically, and 4.75 mm diameter spiral steel was provided horizontally to

reinforce the columns. The spiral spacing was 75 mm along the main column length, and decreased to 25 mm in the end regions within a distance of 250 mm from both ends to minimize end-region stress concentrations. Two CFRP layers with a length of 125 mm were also applied to the end regions, strengthening them. It is important that the same CFRP configuration as that used for the experimental specimen in [16] be used for the specimen being considered for validation. The numerical and experimental responses of the current axial load-displacement case were compared to determine the suitability of the adopted constitutive models, element formulations, interaction assumptions, mesh and boundary conditions (Fig. 2).

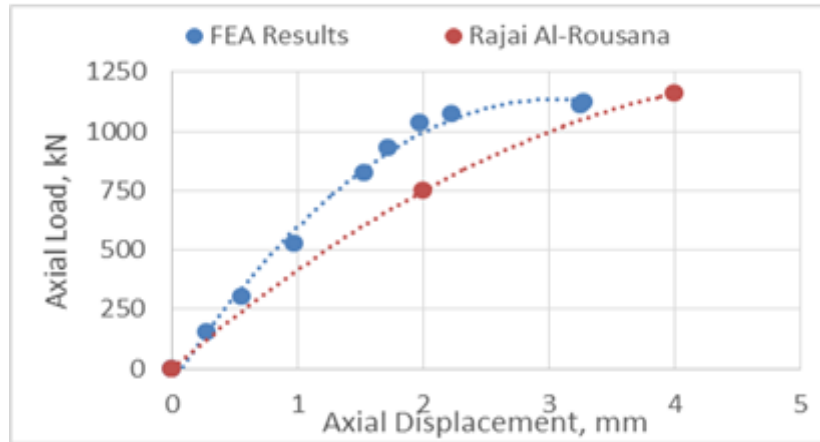


Fig. 2. Comparison between experimental and FEA axial load-displacement responses

The FE model results showed a maximum axial load of around 1120 kN as against that of the experimental result of around 1160 kN, which was 3.45% different. This close agreement suggests that the model is able to give a good prediction of the ultimate axial load bearing capacity. The numerical analysis and experimental data showed that the displacement at peak load was about 3.25 mm and 4.00 mm, respectively, which was about a 18.75% difference (Table VIII).

TABLE VIII. QUANTITATIVE VALIDATION OF THE FINITE ELEMENT MODEL

Validation parameter	Experimental [16]	FEA	Difference (%)
Maximum axial load (kN)	1160	1120	3.45
Axial displacement at maximum load (mm)	4.00	3.25	18.75

The larger difference in the prediction of displacement can be explained by the idealizations in the numerical model, such as the concrete constitutive representation, the assumption of the bond condition between the concrete and reinforcement, the treatment of the interface between the CFRP and the concrete, the idealization of boundary conditions, and the normal variation of properties, measurement, and in the experiments. However, the overall response of the model was found to be a very nonlinear load-displacement response and showed a very small error in predicting the peak load; it was thus concluded that the model was sufficiently accurate for the numerical and parametric study that followed.

The CFRP confinement contribution has been calculated based on the ACI PRC-440.2-23 code for an independent code-based benchmark (Table IX) [2]. The shape efficiency factors are assumed to be $Ka = Kb = 1.0$ for a circular section. The formula below is the response of a section that is continuously confined by the number of transverse CFRP layers indicated. For a direct comparison with the global column capacity, the transverse CFRP configuration has to be the same for the whole of the test region.

TABLE IX. MATERIAL AND GEOMETRIC PROPERTIES USED IN THE ACI PRC-440.2-23 CALCULATION

Parameter	Value
Concrete compressive strength, f'_c	55 MPa
Column diameter, D	150 mm
CFRP elastic modulus, E_f	228 GPa
CFRP thickness per layer, t_f	0.165 mm
Number of CFRP layers, n	2
CFRP ultimate tensile strain, ϵ_{fu}	0.017

Parameter	Value
Environmental reduction factor, CE	0.95
FRP strain efficiency factor, $\kappa\epsilon$	0.55
Strength reduction factor for confinement model, ψf	0.95
Steel yield strength, f_y	413 MPa
Longitudinal reinforcement	4 bars $\times$ 9.5 mm

- Effective CFRP strain

$$\epsilon_{fu} = CE \epsilon_{fu} * \dots\dots\dots(1)$$

$$\epsilon_{fe} = \kappa\epsilon \epsilon_{fu} \dots\dots\dots(2)$$

- Effective lateral confining pressure

$$f_l = 2 E f_n t f \epsilon_{fe} / D \dots\dots\dots(3)$$

- Confined concrete compressive strength

$$f'_{cc} = f'_c + \psi f (3.3) \kappa a f_l \dots\dots\dots(4)$$

ACI PRC-440.2-23 also requires the ultimate axial strain of the confined concrete to be checked. Taking $\epsilon_c' = 0.002$ and $\kappa b = 1.0$ for the circular section:

$$\epsilon_{ccu} = \epsilon_c' [1.50 + 12 \kappa b (f_l/f'_c)(\epsilon_{fe}/\epsilon_c')^{0.45}] \dots\dots\dots(5)$$

Because the calculated $\epsilon_{ccu} = 0.01061$ exceeds the ACI limit of 0.01, the maximum usable concrete strain is limited to 0.01. Using the ACI confined-concrete stress-strain relationship at $\epsilon_c = 0.01$ gives an adjusted confined compressive stress of approximately 81.34 MPa for this analytical comparison, rather than using 82.94 MPa without the strain-limit check. The corresponding confined-strength increase relative to the unconfined concrete is approximately 47.9%.

- Sectional concentric nominal axial strength

$$P_0 = 0.85 f'_{cc,adj} (A_g - A_{st}) + f_y A_{st} \dots\dots\dots(6)$$

$$P_0 = 1,319 \text{ kN}$$

The sectional value using ACI PRC-440.2-23 is about 13.7% higher than the experimental peak load (1160 kN), and about 17.8% higher than the FEA peak load (1120 kN). The difference does not in itself indicate poor FE validation, as the ACI expression is a simplified confinement expression for a design, whereas the experiment and FE analysis actually represent the entire specimen, including end-region effects, material nonlinearity, and actual CFRP configuration. Calculation results are presented in (Table X).

TABLE X. ACI PRC-440.2-23 SECTION-LEVEL SENSITIVITY TO THE NUMBER OF CONTINUOUS TRANSVERSE CFRP LAYERS

CFRP layers	f_l (MPa)	f_l/f'_c	f'_{cc} (MPa)	ϵ_{ccu}	P_0 (kN)
0	0.00	0.000	55.00	0.00200	930
1	4.46	0.081	68.97	0.00680	1136
2	8.91	0.162	82.94	0.01061	1319
3	13.37	0.243	96.90	0.01441	1360
4	17.82	0.324	110.87	0.01821	1383

The table X is included as a sensitivity check for continuous transverse wrapping. For $n \geq 2$, the raw strain prediction exceeds 0.01; therefore, the ACI strain cap affects the usable confined response and prevents the sectional axial strength from increasing linearly with the number of CFRP layers.

6. RESULTS AND DISCUSSION

6.1 Influence of the number of column sides wrapped with CFRP

The higher the number of CFRP-wrapping sheets, the less is the vertical displacement of the column. Columns wrapped on additional surfaces have greater confinement, thereby making them have better stiffness and less axial deformation. The incorporation of CFRP sheets increases the ductile behaviour of the column and enables it to sustain more deformations before it fails. Complete wrap is the one with maximum displacement at maximum loading. It is also due to high confinement and indicates a higher maximum plastic strain.

(Figure 3, a) indicates that the column wrapped with four sides using CFRP sheets had a maximum displacement that exceeded 1.3 times that of the RC column. The axial load-bearing capacity of a concrete column is increased by the introduction of additional sheets of CFRP (Carbon Fiber Reinforced Polymer), which increases the lateral confinement

pressure on the concrete core. The maximum load of a 4-sided wrapped column was 1.34 times that of the RC column, as indicated in (Figure 3, b). Meanwhile, the highest axial stress is subjected to a significant increase. This is because the core concrete is confined due to the lateral confinement. The ultimate axial stress in a 4-sided wrapped column is 1.37 times greater than that of an RC column, as determined in (Figure 3, c). It is found that the 3-sided wrapped columns experienced the least plastic strain, indicating the obvious effect of CFRP sheets on reducing plastic strain (Figure 3, d).

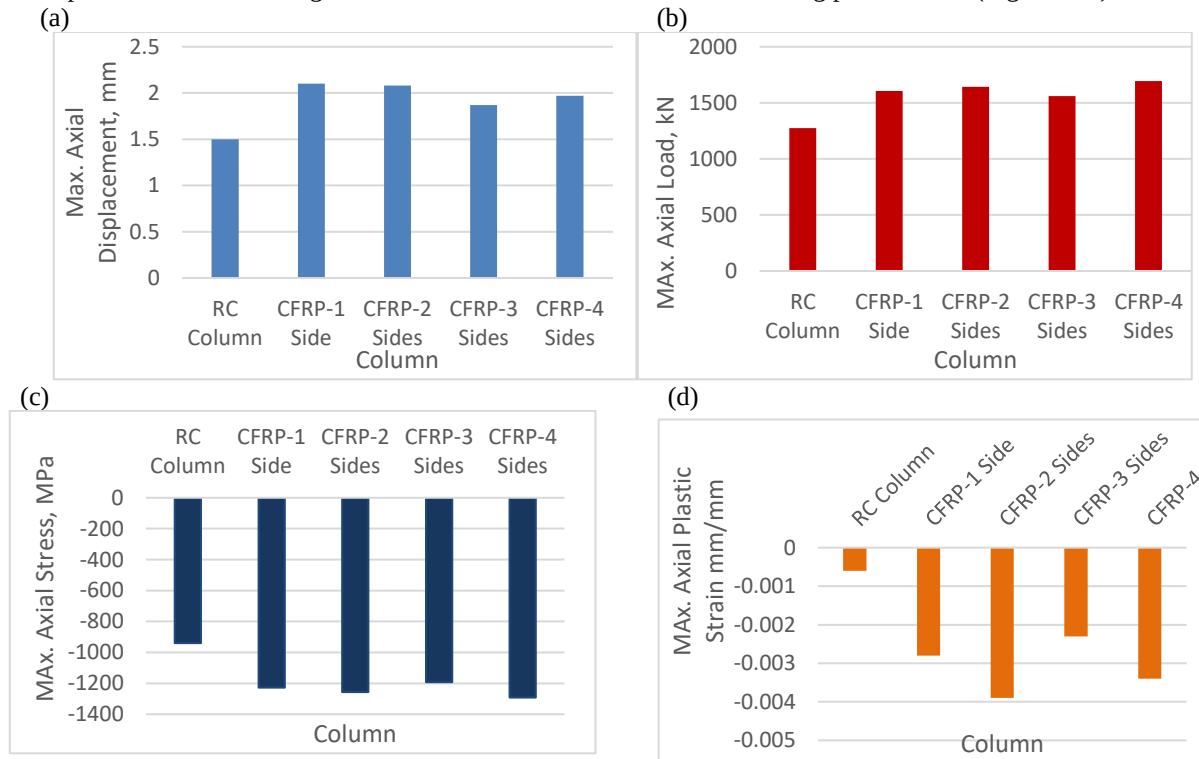


Fig. 3. FEA results for columns: a) maximum displacements, b) maximum load, c) maximum axial stresses, and d) maximum axial plastic strain.

The axial response of the reference RC column and columns reinforced with one, two, three, and four-sided CFRP applications is compared in Figure 3. The results show that the more sides the CFRP is attached to, the higher the axial load capacity and the greater the resistance to compressive stress, except that the deformation response is different when the CFRP is attached to different sides.

The displacement in the RC column is approximately 1.50 mm, and in the CFRP-1, CFRP-2, CFRP-3 and CFRP-4 columns, the displacements are found to be approximately 2.10, 2.10, 1.85 and 1.95 mm, respectively. The relative increase in the displacement of the reinforced columns is linked to their capacity to carry larger axial loads up to the ultimate response. The CFRP not only enhances the elastic stiffness but also affects the post-elastic response by restraining concrete deformations and enhancing the interaction between the CFRP and concrete.

As shown in Fig. 3(b), the maximum axial load increases drastically after CFRP strengthening. The reference RC column reaches about 1250 kN, while the two columns with CFRP, CFRP-1 and CFRP-2, reach about 1600 kN and 1630 kN, respectively, and the two columns with CFRP-3 and CFRP-4 attain 1550 kN and 1680 kN, respectively. The contribution of CFRP-1 is significant since a single-sided application of CFRP gives tensile reinforcement and some lateral compression to the concrete. If two or three sides are reinforced, the confinement will be more effective as more of the column perimeter will be restrained. The fully wrapped CFRP-4 configuration has the greatest axial capacity because it gives full confinement around the column, resulting in a more uniform restriction of the column lateral expansion and hence enhancing the contribution of the concrete core to the column axial capacity. Failure/termination criterion is depended to determine ultimate load when ANSYS non-convergence occurs.

A similar improvement is seen in the maximum axial stress obtained from the complete finite element model as illustrated in Fig. 3(c). The RC column's stress is around -950 MPa, and the CFRP-strengthened columns have higher stress magnitudes. CFRP-1, CFRP-2, CFRP-3, and CFRP-4 reach approximately -1200, -1250, -1180, and -1280 MPa, respectively. The negative values represent compression. The superior stress capacity of the CFRP-strengthened columns is largely attributed to the so-called 'confinement effect' of the CFRP that delays concrete deterioration, which allows the

column to sustain higher compressive stresses. The best stress magnitude found for CFRP-4 is the same as the stress of the complete circumferential confinement.

As can be seen in Figure 3(d), the plastic strain varies significantly depending on the configuration of the CFRP. The maximum axial plastic strain of CFRP-2 is -0.004 , the largest of all the materials, meaning that it has the most potential inelastic capacity. The plastic strain values of CFRP-1 and CFRP-3 are, on the contrary, lower values. That means it is not always possible to obtain a monotonic increase in plastic strain with the addition of strengthened sides. Rather, this redistribution of stresses and occurrence of localized plastic deformation depends on the geometry and extent of the CFRP. Overall, the results show that the four-sided CFRP wrapping (CFRP-4) gives the best results in terms of axial load and stress capacity. This is due to the constant area of confinement all the way around the column perimeter. The results also show that the enhancement efficiency is not only related to the presence of CFRP but also significantly related to the degree of concrete confinement, stress redistribution, and interaction between the CFRP and concrete, which are related to the coverage and configuration of CFRP. Use of CFRP helps in minimizing the vertical displacement of the column under loading, which means that the entire stiffness increases. The wrapping of concrete by CFRP makes the core of the concrete strengthened and therefore restricts axial deformation and also postpones cracking.

When the axial load $P=270$ kN is applied, with increasing number of CFRP sheets, displacement was compared; it was observed that the axial displacements were reduced with an increase in the number of sheets. The confinement effect of CFRP sheets resulted in minimal displacement in the 4-sided wrapped column, as shown in Figure 4.

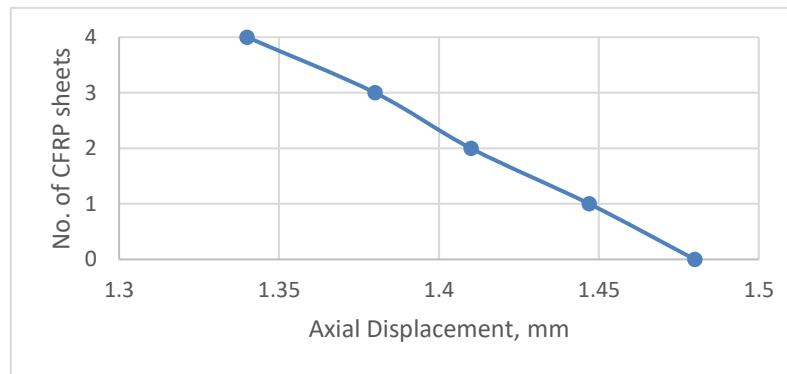


Fig. 4. Axial displacements at axial load, $P = 270$ kN

6.2 Influence of the modulus of elasticity of CFRP sheets

A widely used equation for the lateral confining pressure (hoop action of wrap) according to ACI 440.2R [2] is:

$$f_l = \frac{2tf E_f \epsilon_{fe}}{D} \dots \dots \dots (7)$$

Where: tf = total CFRP thickness (mm), E_f = CFRP elastic modulus ($\text{N/mm}^2 = \text{MPa}$), ϵ_{fe} = effective hoop strain in the CFRP, D = column characteristic dimension (mm).

The column with all four sides wrapped by CFRP and $E = 250$ GPa experienced the greatest displacement. Results in Tables XI-XIII show that the maximum axial load is 1597.5 kN. The CFRP elastic modulus significantly affected the response; however, the relationship was non-monotonic over the investigated range, and the maximum axial stress obtained from the complete finite element model is 1223 MPa. The confinement causes the maximum limits of axial load to increase as it delays crushing and enables the column to achieve a higher ultimate stress.

TABLE XI. FEA RESULTS FOR MAXIMUM DISPLACEMENTS

CFRP Modulus, E (GPa)	Confined Columns – Max. Axial Displacement (mm)	Unconfined Columns – Max. Axial Displacement (mm)
250	1.82	1.5
300	1.38	1.5
350	1.59	1.5
400	1.76	1.5

TABLE XII. FEA RESULTS FOR MAXIMUM AXIAL LOAD

CFRP Modulus, E (GPa)	Confined Columns – Max. Axial Load (kN)	Unconfined Columns – Max. Axial Load (kN)
250	1550	1250
300	1250	1250

350	1450	1250
400	1597.5	1250

TABLE XIII. FEA RESULTS FOR MAXIMUM AXIAL STRESS

CFRP Modulus, E (GPa)	Confined Columns – Max.Axial Stress (MPa)	Unconfined Columns — Max. Axial Stress (MPa)
250	–1200 MPa	–950
300	–1000 MPa	–950
350	–1100 MPa	–950
400	–1223 MPa	–950

The results show that the axial response is significantly affected by the CFRP modulus, but the relationship is not necessarily monotonic, as it depends on the interplay between the stiffness of the CFRP, the confinement of the concrete, and the redistribution of the load.

The maximum axial displacement of the confined columns ranges from 1.38 to 1.82 mm, whereas that of the unconfined one's 1.5 mm (Table XI). It can also be seen that the higher the stiffness of the CFRP, the more the maximum displacement of the strengthened system is reduced, which shows that the initial stiffening capability of the strengthened system is enhanced. The increased CFRP stiffness from 250 GPa to 300 GPa for the confined columns also shows that the maximum displacement is decreased from 1.82 mm to 1.38 mm, which means that the initial stiffening capability of the strengthened system is enhanced. The displacement, however, rises to 1.59 mm and 1.76 mm at 350 GPa and 400 GPa, respectively. This non-monotonic behaviour can be explained by the effect of the interaction between the CFRP layer and the concrete substrate. When the stiffness of the CFRP increases, the stresses and deformations of the composite system change and do not necessarily reduce the global displacement.

From Table XII, The confined columns exhibited higher axial capacities than the RC column, except for the E=300 GPa, where both capacities were approximately 1250 kN. The confined columns reach 1550, 1250, 1450, and 1597.5 kN for CFRP moduli of 250, 300, 350, and 400 GPa, respectively, whereas the corresponding unconfined columns reach 1250 kN. This shows that the contribution of the axial load-carrying capacity of CFRP confinement is large. The CFRP confines the concrete core under compression and thus slows down the formation of critical damage, which improves the lateral confinement of the concrete. It is seen that the highest axial load occurs at a E = 400GPa, the highest response among the investigated modulus values was obtained at 400 GPa, thus suggesting that the stiffness of the CFRP plays a role in the confinement and transfer of stress at this value.

The effect of CFRP strengthening on the maximum axial stress obtained from the complete finite element model is further illustrated using the results in Table XIII. The maximum axial stress obtained from the complete finite element model stresses in the columns under confinement are –1200, –1000, –1100, and –1223 MPa for CFRP moduli of 250, 300, 350, and 400 GPa, respectively. These unconfined columns suffer a smaller stress magnitude of about –950 MPa. The increased stress capacity of the confined columns in stress capacity further confirms that the CFRP increases the resistance of the concrete to lateral deformation and that the concrete – CFRP system can withstand a higher level of stress before attaining its maximum response.

It should also be noted that there is no monotonic response with the increase in CFRP modulus for all parameters. For instance, the maximum displacement and the axial load for the confined case with a modulus of 300 GPa are the lowest for all these confined cases, while that of 400 GPa is the highest. This shows that stiffness of CFRP, the confinement of concrete, and stress redistribution play a role in the mechanical response rather than the CFRP modulus.

The axial displacements of columns with varying CFRP sheet modulus of elasticity values under constant axial load P=270 kN, when compared, show that the displacement reduced with an increase in the E value of the column. (Figure 5) demonstrates that the minimum displacement is noticed at E =400 GPa, which was 5 percent less than that of CFRP sheets at 250 GPa.

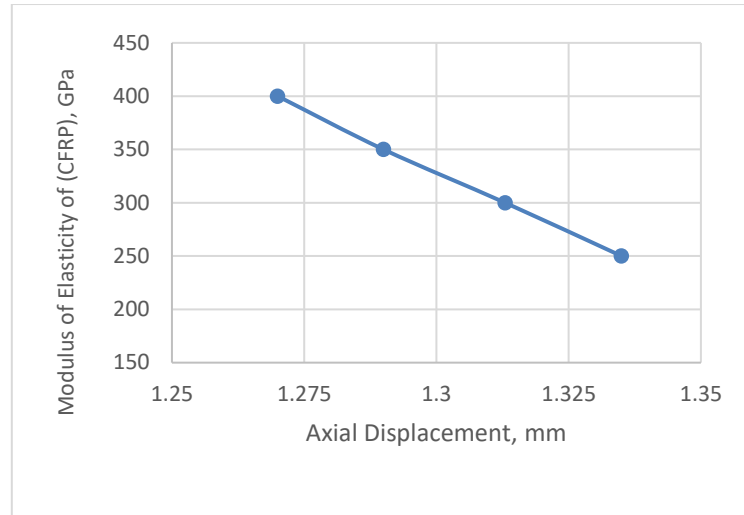


Fig. 5. Axial displacements for different CFRP modulus of elasticity values at Axial load, $P = 270$ kN

6.3 Influence of the thickness of CFRP sheets

The effect of CFRP thickness on the axial response of the columns is shown in Figure 6. The thicknesses of CFRP examined are 0.5, 1.0, 1.5 and 2.0 mm, besides the reference RC column. Based on the results, the thickness of the CFRP generally increases the axial load-bearing capacity and the stress capacity of the strengthened columns.

The axial displacement is found to be 1.50 mm for the reference RC column, while for 0.5, 1.0, 1.5 and 2.0 mm of CFRP thickness, it is 1.63 mm, 1.68 mm, 1.77 mm and 1.56 mm respectively, as shown in Fig. 6(a). The displacement increase to 1.5 mm can be attributed to the strengthening of the column, increasing the capacity of the load being carried by the column and thus increasing the capacity of the column to carry the axial load before reaching the maximum response. The small decrease at 2.0 mm indicates that there is not a direct relationship between CFRP thickness and deformation capacity, and that excess thickness can change the deformation and stress distribution of the composite section.

As is evidenced from Figure 6(b), the maximum axial load increases with CFRP thickness. Approximately 1200 kN is applied to the reference RC column, and approximately 1350, 1480, 1600, and 1600 kN are applied to the strengthened columns with CFRP thicknesses of 0.5 mm, 1.0 mm, 1.5 mm, and 2.0 mm, respectively. This improvement is primarily due to the greater confinement of the thick CFRP layer and the added load transfer ability. Thicker CFRP provides greater hoop stiffness/resisting force and therefore restrains lateral dilation.

The maximum axial stress obtained from the complete finite element model in Fig. 6(c) is similar. The stress magnitude is raised from about -950 MPa for the RC column to about -1050, -1100, -1200 and -1150 MPa for 0.5, 1.0, 1.5 and 2.0 mm-thick CFRPs, respectively. The numerical increase indicates that strengthening the concrete column with CFRP enhances the stress-bearing capacity of the concrete column. Maximum stress is achieved at 1.5 mm thickness, and the increase in thickness to 2.0 mm only slightly lowers the stress, meaning that the thicker the CFRP, the higher the stress only up to a certain point.

The maximum axial plastic strain is shown in Figure 6(d). The plastic strain values are about -0.0007 for the RC column, while they are about -0.0010, -0.0018, and -0.0027 for 0.5, 1.0, and 1.5 mm thick CFRP, respectively. The magnitude is around -0.0016 at 2.0 mm. The 1.5 mm thickness of the CFRP results in a higher inelastic deformation before maximum response is reached, while the 2.0 mm thickness CFRP gives a stiffer response and therefore a lower plastic deformation.

In general, the outcomes suggest that the axial load and stress capacity of the column are improved by increasing the thickness of CFRP. There are, however, no linear relationships. The thickness of 1.5 mm CFRP seems to give the best overall performance, namely high axial load capacity, high stress capacity, and the greatest plastic deformation capacity. Increasing the thickness further to 2.0 mm does not necessarily lead to an improvement, indicating that there is an optimum thickness of CFRP after which further thickness gives a smaller structural advantage. This behavior demonstrates the need to optimize CFRP thickness and not just the amount of strengthening material.

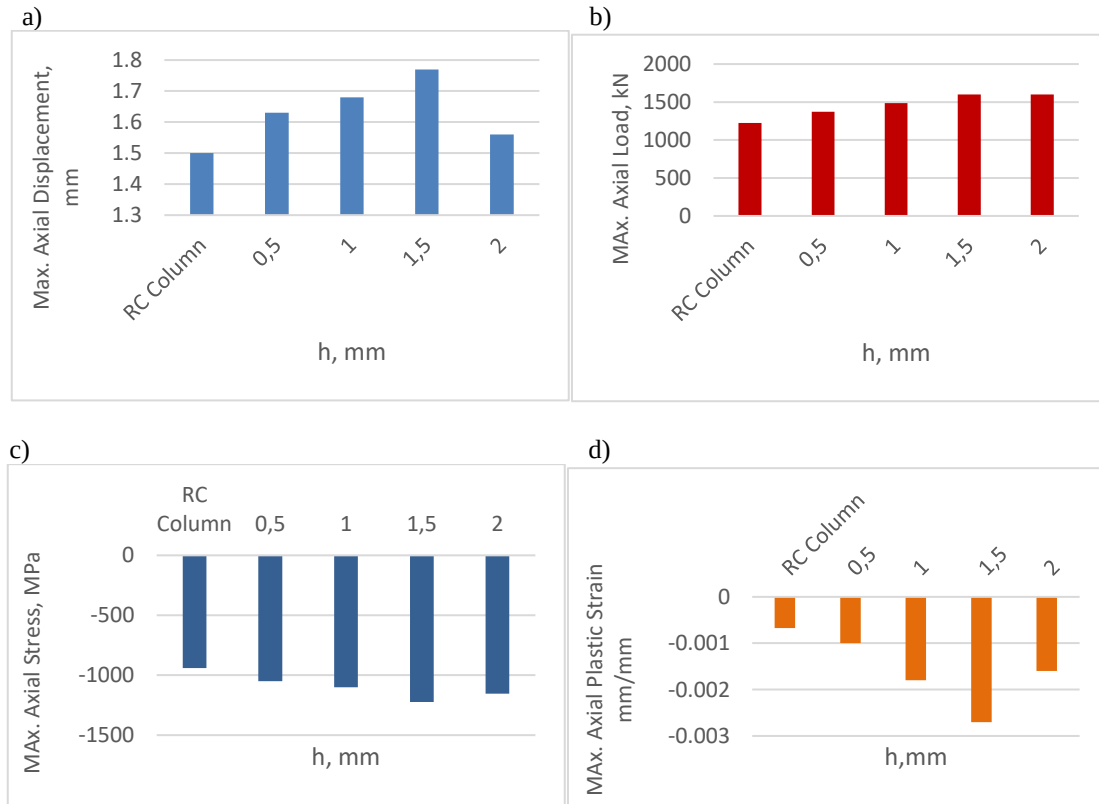


Fig. 6. FEA results for columns: a) maximum displacements, b) maximum load, c) maximum axial stress obtained from the complete finite element model, and d) maximum axial plastic strain.

7. HARMONIC RESPONSE ANALYSIS

Cyclic loads will induce a sustained cyclic response (harmonic response) in any structural system. Harmonic response analysis facilitates the determination of the long-term dynamic behavior of the structural components, e.g., reinforced concrete columns, hence enabling the determination of whether their design will be effective in counteracting resonance or other harmful impact of forced vibration.

The steady-state dynamic response behavior of the reinforced concrete columns was explored using a harmonic response analysis, with sinusoidal excitations. Analysis was performed in a frequency range from 4–44 Hz to examine the variation of the structural response over the range of the excitation frequencies. Structural damping coefficient of 0.05 was proposed. To provide a fair comparison, the same boundary conditions, loading conditions, material properties, and excitation amplitude were used for the analysis of the reference RC column and the four-sided CFRP-strengthened column.

The maximum stress obtained from the complete finite element model (horizontal stress, longitudinal stress), axial displacement, and frequency-dependent structural response were used as criteria to evaluate the harmonic response. A series of discrete frequencies were excited, and results were obtained for each one of these frequencies within the investigated range.

The harmonic loading has a peak amplitude 5000 kN, with frequency range of substep 4–44. Compared to discrete mathematics, the numerical analysis is an approximation method, which is used in this work in the form of the finite element approach (FEA) in ANSYS to model the reinforced concrete columns. The FEA used by ANSYS to model the composite columns is based on the correct choice of the type of elements; in particular, SOLID65 as the type of concrete elements and SOLID45 as the type of steel plate at the top of the column, which is used to scatter the load applied. Other than the element types, number of elements, suitable material models, and boundary condition simulation of the column at the ends, convergence criteria and solution methods are selected. The Newton-Raphson method was applicable in the same analysis as the static analysis. A modal analysis is used to determine frequencies within the resonance range of the model, then the dynamic analysis commences. All the reinforced concrete columns are built based on the ANSYS APDL program to

accomplish the analysis of both the static and harmonic dynamic analyses. The full harmonic equations were solved using the sparse direct solver.

Figures 7 and 8 indicate the difference in the constituents of stress with the excitation frequency of both conventional reinforced concrete (RC) and the RPCF-4 sheet specimen. The structural response can be considered to be very frequency dependent. In this case, the horizontal stress of the RC is much higher than that of the CFRP-4-sided column, as seen in Fig. 7, at the lowest frequency investigated, namely 170 N/mm². The horizontal stress of the RC column is seen to rapidly decrease and reach a very small value as the frequency increases. The horizontal stress remains low throughout the frequency range investigated in the CFRP-strengthened column. The result shows that the 4-sided wrapping with CFRP material has a significant ability to limit lateral deformation and mitigate horizontal stresses under dynamic excitation.

A similar pattern is observed for the longitudinal stress as shown in Figure 8. The RC column develops a strength of around 520 N/mm² at the lowest frequency, while the CFRP-4-sided column has a significantly lower value. The RC column shows a significant drop in stress in the direction of the column as the frequency increases, and the stress becomes close to zero values at higher frequencies. Throughout the frequency range, the stress in the CFRP-strengthened column is very low.

RC will always provide significantly larger values of stress in comparison to RPCF-4 at all frequencies. This is more so at lower frequencies; the RC stresses are orders of magnitude larger than those of RPCF-4. In addition, the stress in the longitudinal-direction (Fig.8) is always higher than the stress in the horizontal direction (Fig. 7).

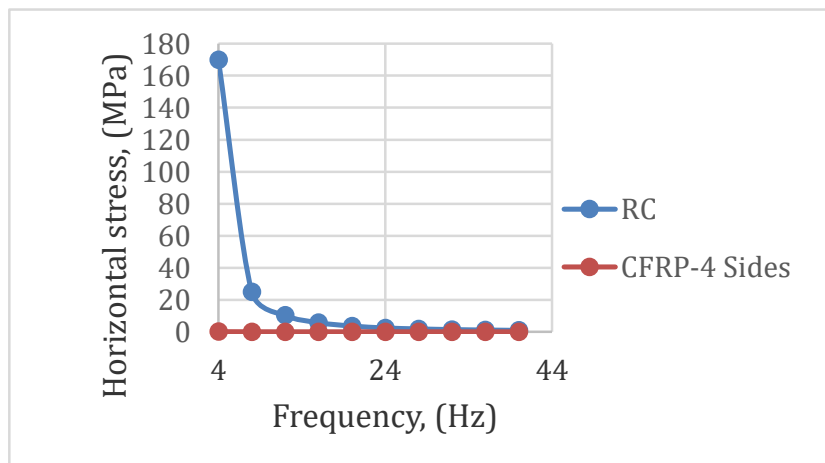


Fig. 7. Horizontal stress for both columns vs. frequency (medium)

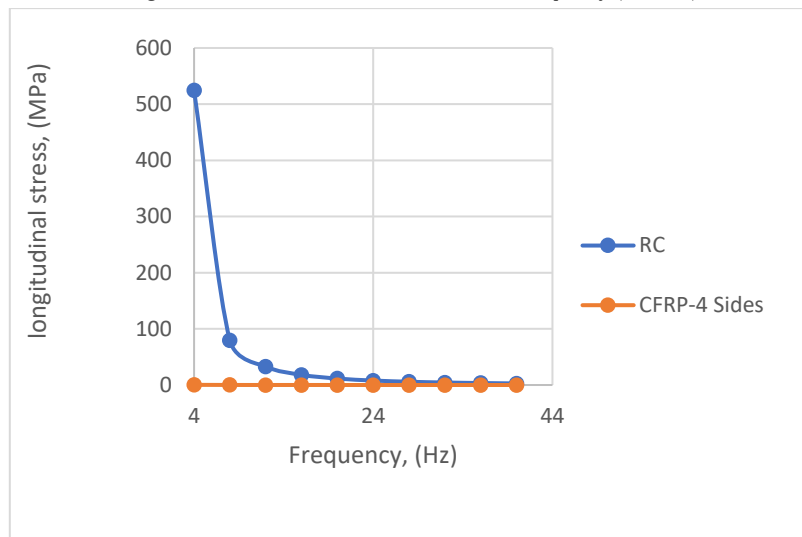


Fig. 8. Longitudinal stress for both columns vs. frequency (medium)

The harmonic response results show that the displacement of the conventional RC and CFRP-strengthened columns is mainly controlled by the X-direction. The maximum UX amplitude was 9.11×10^{-6} at 4 Hz for both models and gradually decreased to 2.26777×10^{-6} at 44 Hz (Tables XIV, XV). The frequency responses of the RC and CFRP-strengthened columns showed an almost identical dominant displacement response in the frequency range of interest.

Thus, under the adopted harmonic loading conditions, the global displacement response of the column in the direction of the excitation was not significantly modified by the CFRP confinement. This indicates that the main advantage of CFRP strengthening could be related to confinement, strength, damage control, and nonlinearity rather than significant changes in the elastic harmonic displacement response in the studied frequency range.

TABLE XIV. HARMONIC DISPLACEMENT RESPONSE OF THE REINFORCED CONCRETE COLUMN AT NODE 735 (TOP CENTRAL NODE)

Frequency (Hz)	UX Amplitude	UX Phase (°)	UY Amplitude	UY Phase (°)	UZ Amplitude	UZ Phase (°)
4	2.51145×10^{-5}	180	3.36150×10^{-24}	0	2.18606×10^{-23}	0
8	1.24732×10^{-5}	180	3.00371×10^{-25}	0	2.82060×10^{-24}	0
12	8.32094×10^{-6}	180	7.90432×10^{-26}	0	8.70210×10^{-25}	0
16	6.23865×10^{-6}	180	5.73926×10^{-26}	0	3.31956×10^{-25}	0
20	4.99016×10^{-6}	180	3.07608×10^{-26}	0	1.90751×10^{-25}	0
24	4.15812×10^{-6}	180	1.29362×10^{-26}	0	9.56605×10^{-26}	0
28	3.56393×10^{-6}	180	1.06182×10^{-26}	0	6.02951×10^{-26}	0
32	3.11833×10^{-6}	180	5.65456×10^{-27}	0	4.65842×10^{-26}	0
36	2.77179×10^{-6}	180	5.63856×10^{-27}	0	2.94347×10^{-26}	0
40	2.49457×10^{-6}	180	2.84314×10^{-27}	0	2.19007×10^{-26}	0
44	2.26777×10^{-6}	180	2.56502×10^{-27}	0	1.71922×10^{-26}	0

TABLE XV. HARMONIC DISPLACEMENT RESPONSE OF THE CFRP-STRENGTHENED RC COLUMN AT NODE 735 (TOP CENTRAL NODE)

Frequency (Hz)	UX Amplitude	UX Phase (°)	UY Amplitude	UY Phase (°)	UZ Amplitude	UZ Phase (°)
4	2.51145×10^{-5}	180	1.53333×10^{-15}	180	2.97683×10^{-15}	180
8	1.24932×10^{-5}	180	4.57629×10^{-17}	180	9.63584×10^{-17}	180
12	8.32094×10^{-6}	180	5.98087×10^{-18}	180	1.26506×10^{-17}	180
16	6.23865×10^{-6}	180	1.41560×10^{-18}	180	1.29761×10^{-18}	180
20	4.99016×10^{-6}	180	4.63307×10^{-19}	180	9.81533×10^{-19}	180
24	4.15812×10^{-6}	180	1.86072×10^{-19}	180	3.94293×10^{-19}	180
28	3.56393×10^{-6}	180	8.60552×10^{-20}	180	1.82379×10^{-19}	180
32	3.11834×10^{-6}	180	4.41273×10^{-20}	180	9.35282×10^{-20}	180
36	2.77179×10^{-6}	180	2.44833×10^{-20}	180	5.18956×10^{-20}	180
40	2.49457×10^{-6}	180	1.44553×10^{-20}	180	3.06413×10^{-20}	180
44	2.26777×10^{-6}	180	8.97480×10^{-21}	180	1.90247×10^{-20}	180

In general, the results support the hypothesis that with the rise of the excitation frequency, it is possible to drastically reduce the stress and that RPCF-4 has an important impact on the dynamic performance of the RC column. These results illustrate the suitability of RPCF-4 as an excellent material to be used in processes that require dynamic or vibration loads like vibration-dampening systems.

8. CONCLUSIONS

The present numerical study assessed the static and harmonic load behaviour of RC columns strengthened with CFRP. Based strictly on the numerical results obtained in this study, the following conclusions can be drawn:

1. The investigated RC columns were strengthened by the CFRP method, and the load-carrying capacity in the axial direction increased. The four-sided CFRP configuration had the highest axial load (1680 kN) compared to the reference RC column (1250 kN), which is about 34% higher. The capacities of maximum axial stress obtained from the complete finite element model stress were also the highest for the four-sided configuration.
2. The CFRP coverage configuration had a tremendous impact on the response. The additional wrapped sides did not result in a perfect monotonic change in displacement or plastic strain. Thus, the strengthening efficiency is not only related to the number of CFRP covered sides, but also related to the continuity and distribution of confinement.
3. Generally, the thickness of the CFRP was increased to improve the axial load capacity up to around 1.5 mm. The maximum load was around 1600 kN at 1.5 mm and 2 mm, showing that, beyond a thickness of 1.5 mm, there is little further increase in the axial capacity. The 1.5 mm configuration also showed maximum stress and plastic-strain magnitude of the various thicknesses studied.
4. CFRP elastic modulus had a non-monotonic influence. The highest axial load, around 1597.5kN, and the highest stress magnitude, around 1223 MPa, were obtained for the 400GPa case among the values investigated. However, the intermediate modulus values were not proportional in terms of load, displacement, or stress. Therefore, the numerical findings confirm the strong effect of the stiffness of the CFRP, but not an obvious linear correlation between the elastic modulus and the structural behavior.
5. For the case of harmonic excitation, the four-sided CFRP-strengthened model had lower levels of horizontal and longitudinal stresses compared to the reference RC model, especially at the lower frequencies investigated. The

displacement amplitude in the X-direction, however, was nearly the same for both the RC and CFRP-strengthened models within the range of frequencies tested. Hence, the present results provide evidence of stress redistribution and reduction.

6. The FE model was validated against published experimental results, yielding an ultimate axial load of around 1120 kN, which is 3.45% different from results obtained from the experiments and is considered to be in good agreement. The difference in displacement at the maximum loading was about 18.75%. The results suggest that the model is capable of providing satisfactory prediction of the ultimate axial capacity, and the predictions of deformations show relatively greater disparities.
7. 4-sided CFRP confinement and CFRP thickness of ~1.5 mm resulted in the best static performance within the parameters studied. The results presented here are valid only for the adopted geometry, material characteristics, loading, and the numerical assumptions made, and more experimental evidence and studies are needed to develop comprehensive design guidelines.

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Conflicts of Interest:

The authors declare that there are no competing interests associated with this work.

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